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Center for Integrated Computation and Analysis of Reconnection and Turbulence

The Plasmoid Instability in High-Lundquist-Number Plasmas: Dynamics and Statistics

A. Bhattacharjee, Y-M. Huang, and B. Sullivan

Center for Integrated Computation and Analysis of Reconnection and Turbulence (CICART)

Center for Magnetic Self-Organization (CMSO)

University of New Hampshire

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Dedicated to Masaaki Yamada-San on his 70th Birthday

- I have had the privilege of knowing Masaaki from 1977 when I began as a graduate student at Princeton.
- Through a broad range of experiments in spheromaks, tokamaks, and MRX (too many to be reviewed here), Masaaki's work has done much to illuminate the concepts of magnetic relaxation and reconnection in plasmas, both in collisional as well as collisionless regimes.
- He has been known to treat theorists with a healthy combination of skepticism and grudging respect. (Having Russell Kulsrud around surely helps the theorist's cause.)
- I look forward to his continued involvement and leadership in demolishing some old theories and inspiring new ones.

Sweet-Parker (Sweet 1958, Parker 1957)

Geometry of reconnection layer : Y-points (Syrovatsky 1971)

Length of the reconnection layer is of the order of the system size >> width Δ

Reconnection time scale

$$\tau_{SP} = \left(\tau_A \tau_R\right)^{1/2} = S^{1/2}\tau_A$$

Solar flares: $S \sim 10^{12}, \quad \tau_A \sim 1s$

$$\Rightarrow \tau_{SP} \sim 10^6 s$$ Too long!

Onset of fast reconnection in large, high-Lundquist-number systems

- As the original current sheet thins down, it will inevitably reach kinetic scales, described by a generalized Ohm's law (including Hall current and electron pressure gradient).
- A criterion has emerged from Hall MHD (or two-fluid) models, and has been tested carefully in laboratory experiments (MRX at PPPL, VTF at MIT). The criterion is:

  $\delta_{SP} < d_i$ (Ma and Bhattacharjee 1996, Cassak et al. 2005) or $\delta_{SP} < \rho_s$ in the presence of a guide field (Aydemir 1992; Wang and Bhattacharjee 1993; Kleva, Drake and Waelbroeck 1995)

Transition from Collisional to Collisionless Regimes in MRX

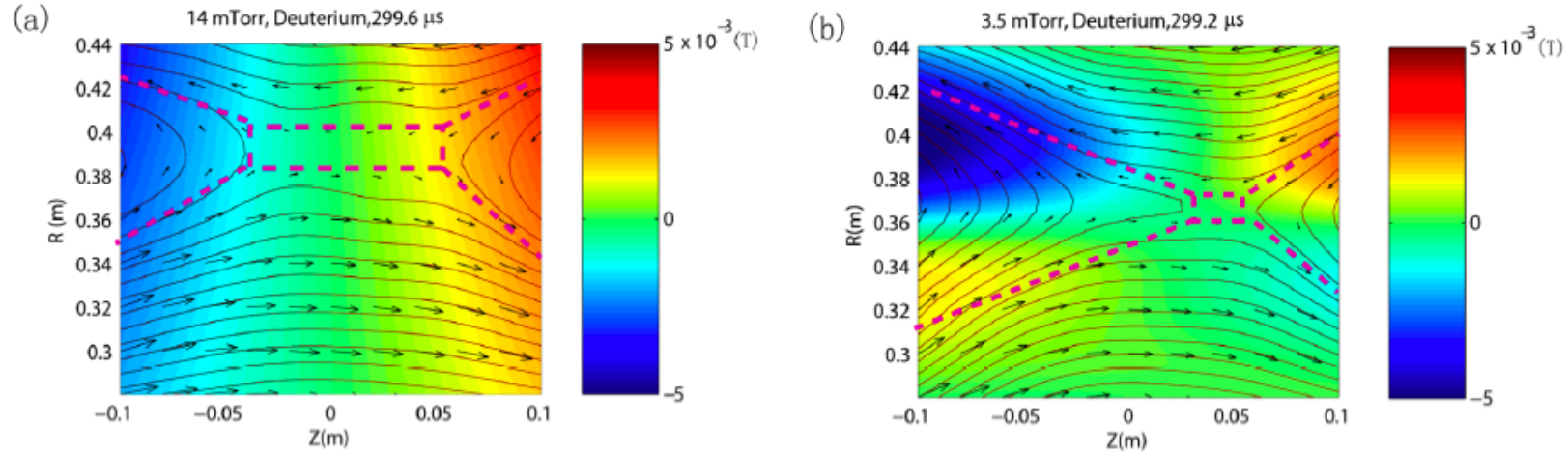
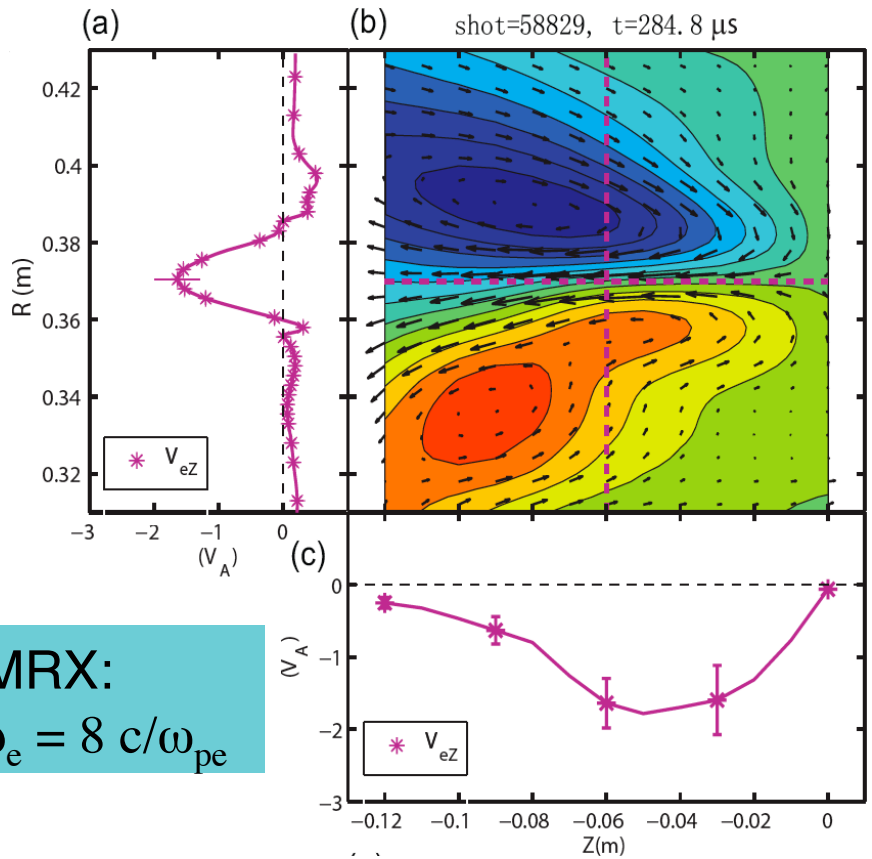
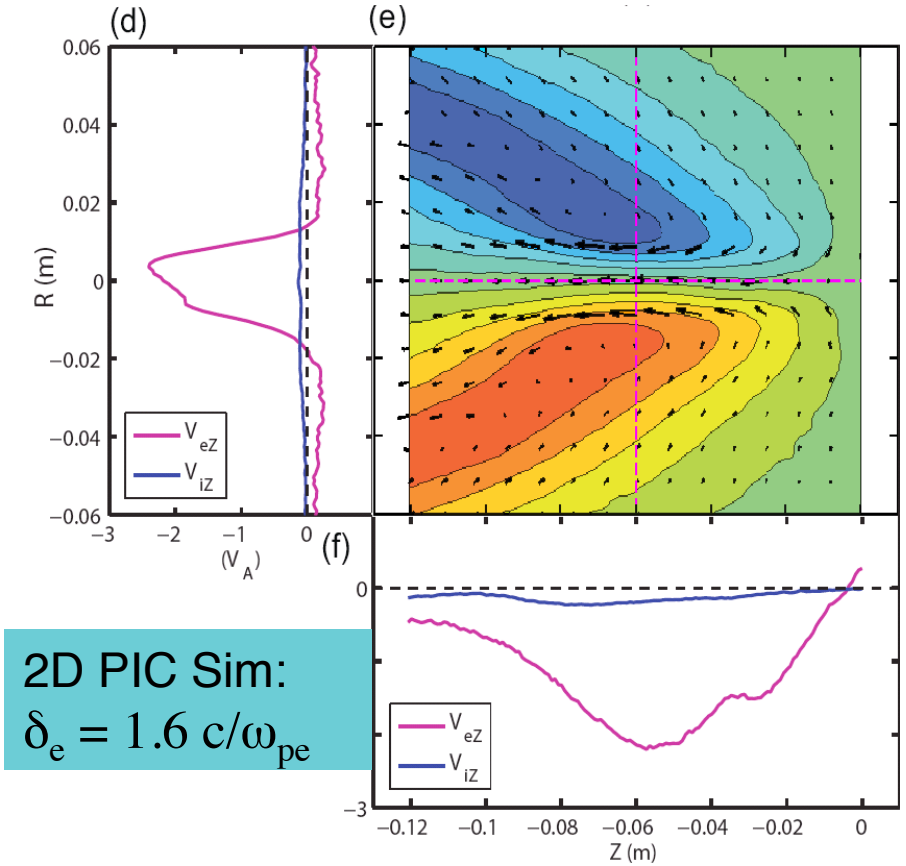


***Fig.2**; Comparison of neutral sheet configuration described by measured magnetic field vectors and flux counters for high (collisional) and low density cases; (a) Collisional regime (λ_{mfp} ~1mm << δ)); (b) Nearly collisionless regime (λ_{mfp} ~1cm ~δ). Out-of plane fields are depicted by the color codes ranged -50 G <Bt <50 G.*

Investigation of Electron Diffusion Layer Made in MRX: Comparison with 2D PIC Simulations





MRX:
$\delta_e = 8\ c/\omega_{pe}$

2D PIC Sim:
$\delta_e = 1.6\ c/\omega_{pe}$

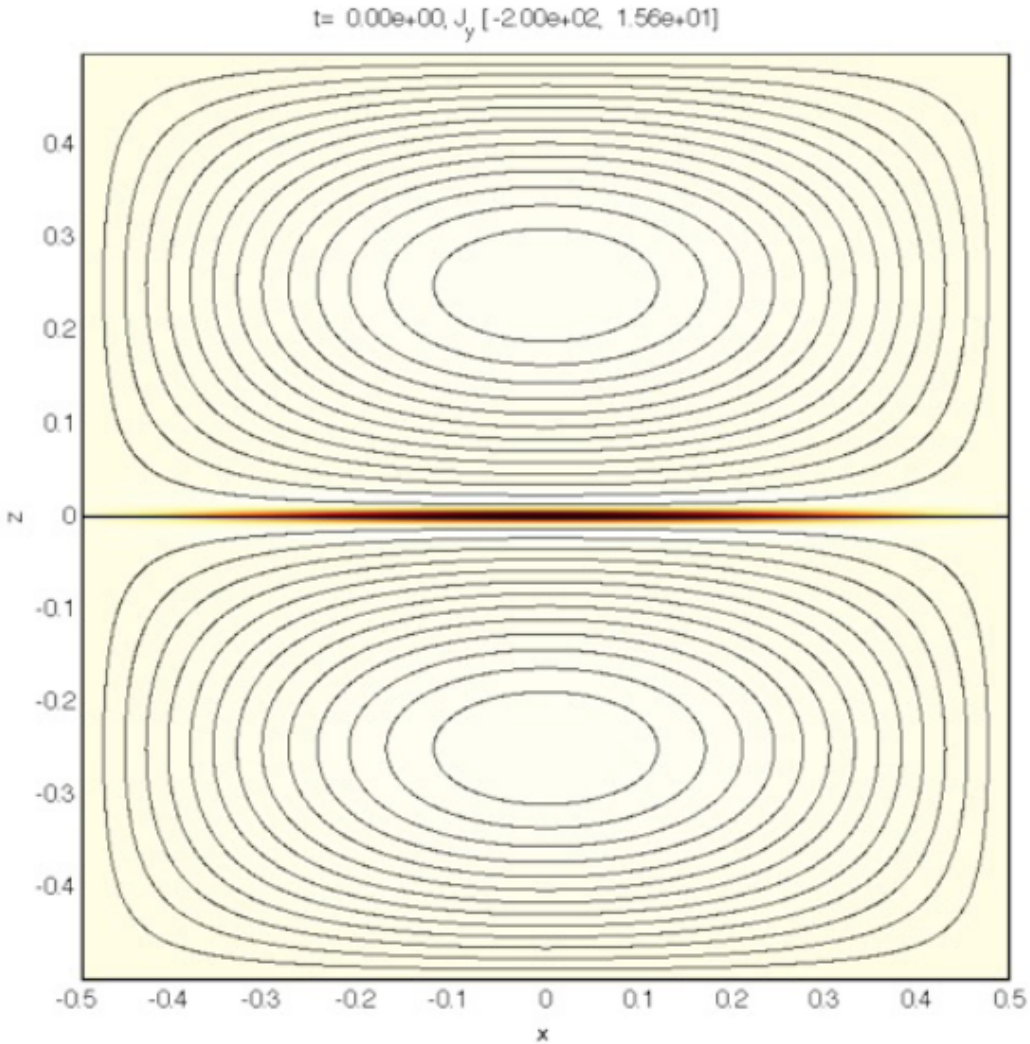
All ion-scale features reproduced, but electron-layer is 5 times thicker in MRX. Importance of 3D effects?

Fast Reconnection in Large Systems

- Extended thin current sheets of high Lundquist number are unstable to a super-Alfvenic tearing instability (Loureiro et al. 2007), which we call the "plasmoid instability," because it generates a large number of plasmoids.
- In the nonlinear regime, the reconnection rate becomes nearly independent of the Lundquist number, and is much larger than the Sweet-Parker rate.

A little history

- Secondary tearing instability of high-S current sheet has been known for some time (Bulanov et al. 1979, Lee and Fu 1986, Biskamp 1986, Matthaeus and Lamkin 1986, Yan et al. 1992, Shibata and Tanuma 2001), but its precise scaling properties were determined only recently.
- The instability has been studied recently nonlinearly in fluid (Lapenta 2008, Cassak et al. 2009; Samtaney et al. 2009) as well as fully kinetic studies with a collision operator (Daughton et al. 2009).



A Low Amplitude Random Forcing is Added

$$\partial_t(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) = -\nabla p - \nabla\psi\nabla^2\psi + \epsilon\mathbf{f}(\mathbf{x},t)$$

$$\langle f_i(\mathbf{x},t) f_j(\mathbf{x}',t')\rangle \sim \delta_{ij}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

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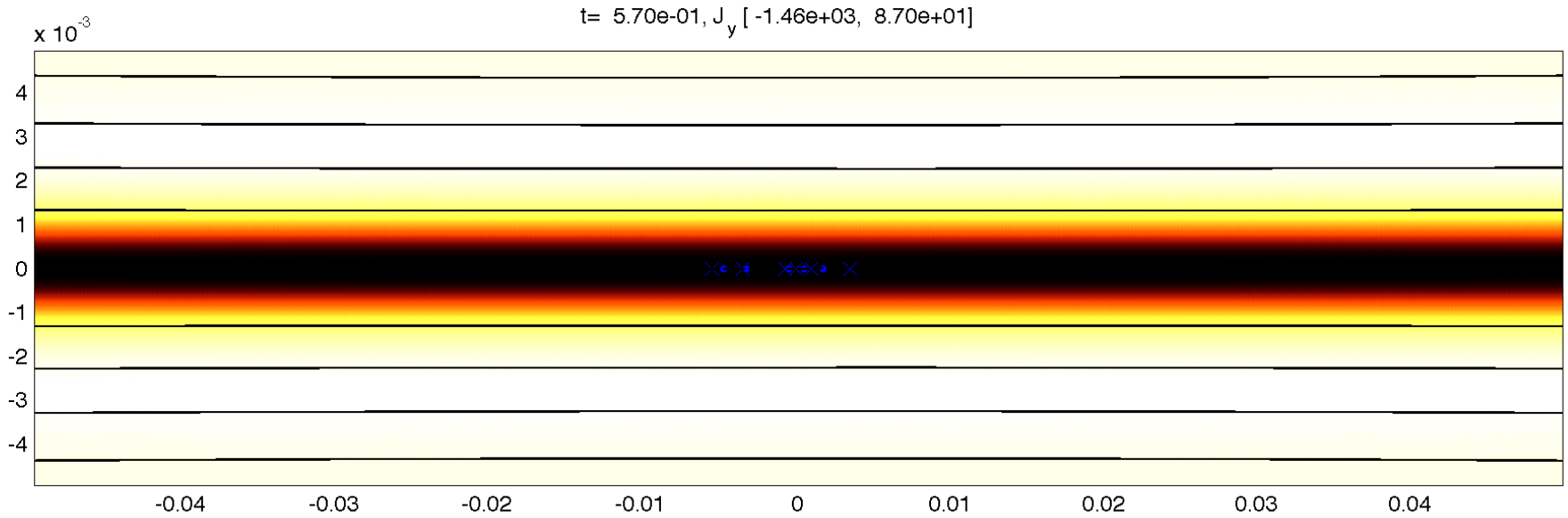
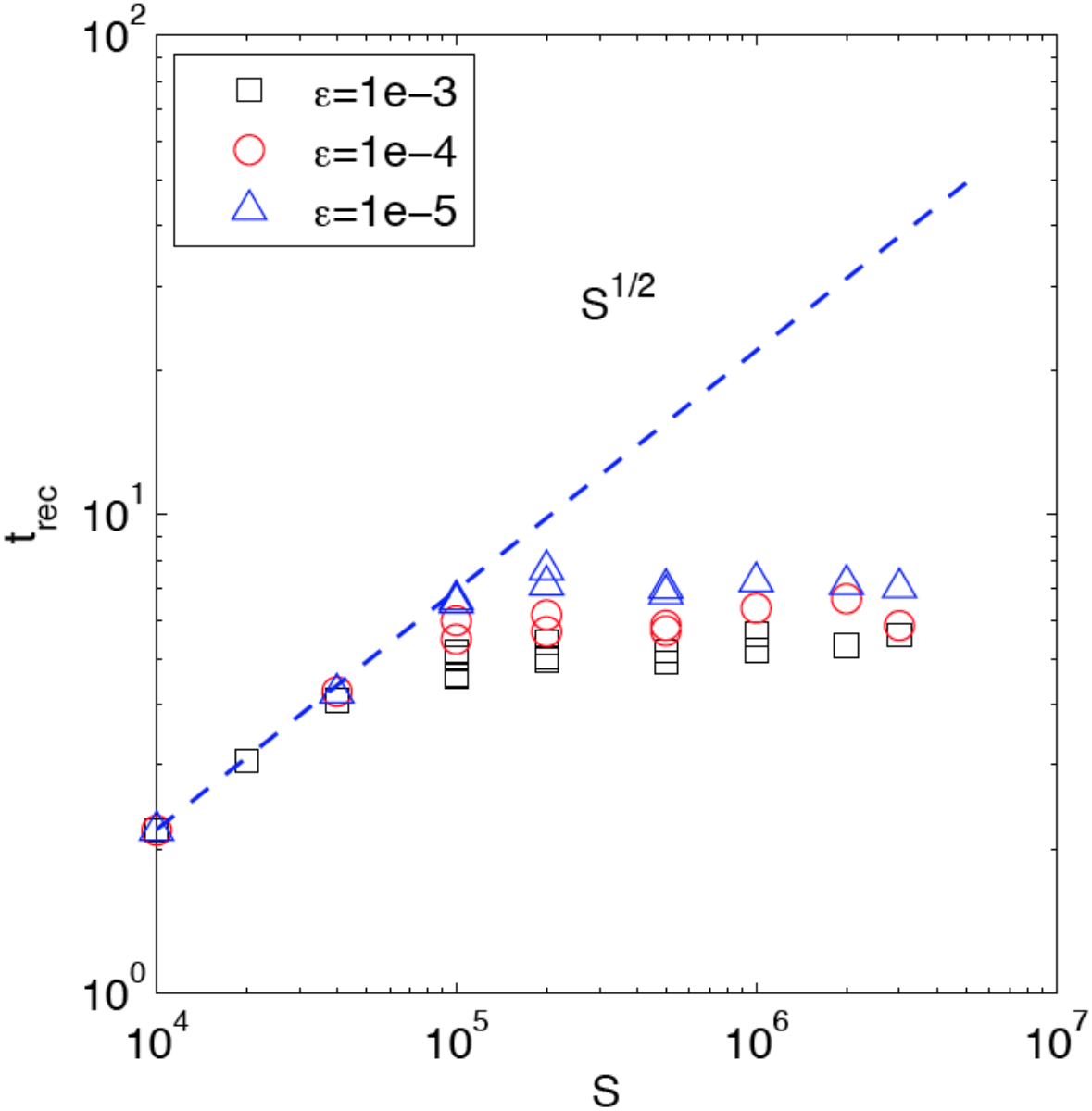


Reconnection Time of 25% of Initial Flux

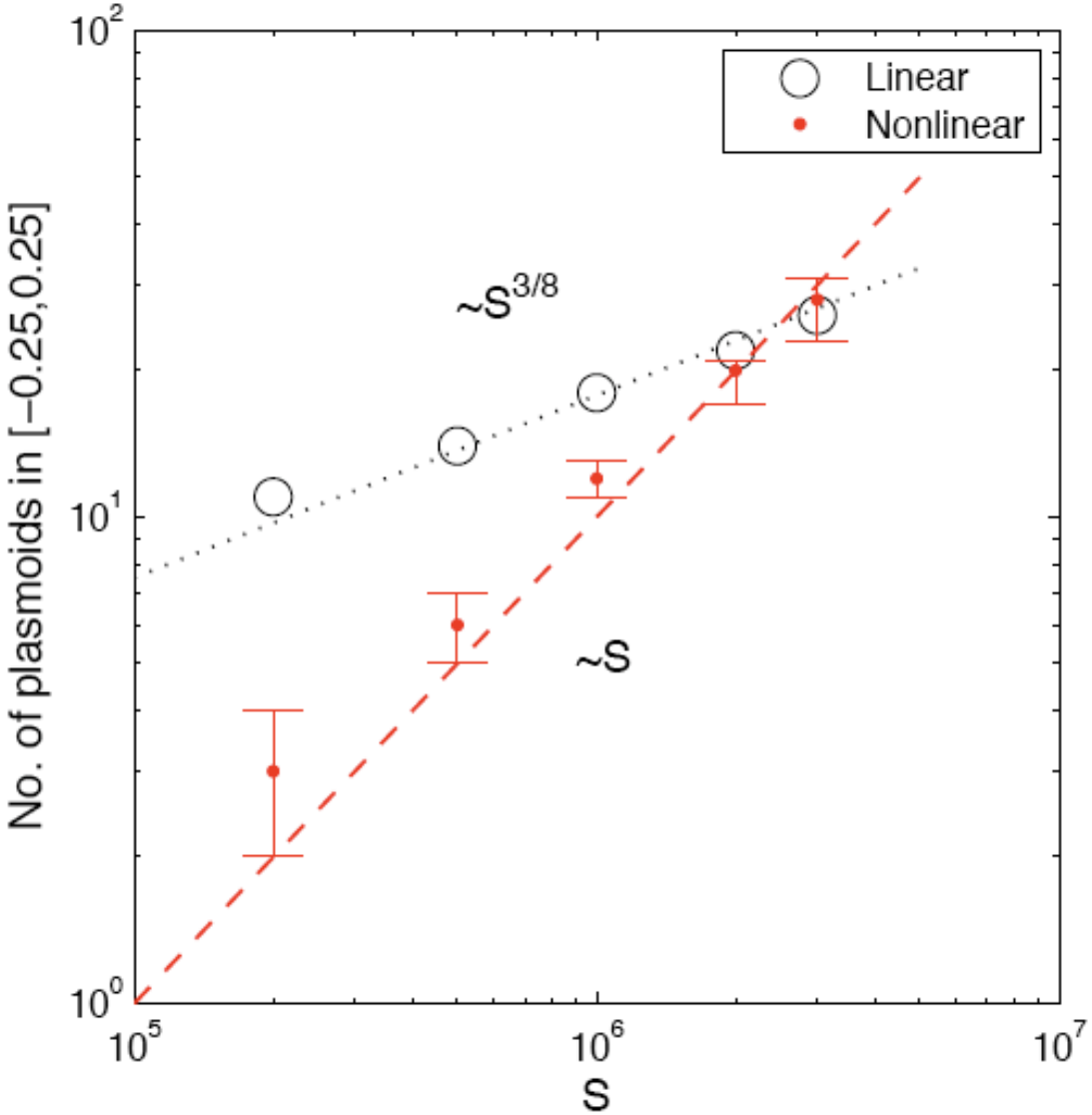


$$< \frac{1}{V_A B} \frac{d\psi}{dt} > \sim 0.01$$

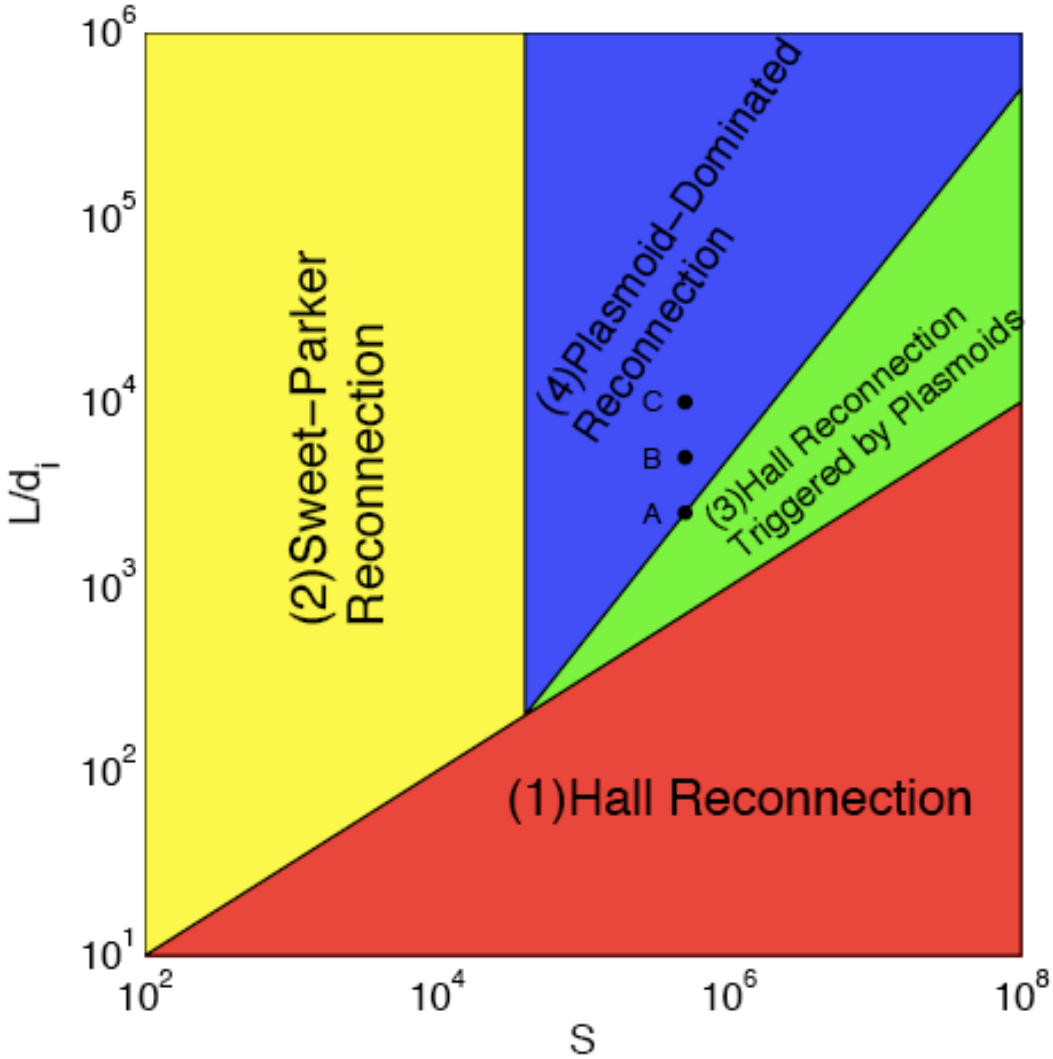
$$< u_i > \sim 0.01 V_A$$

Scaling: heuristic arguments

- Critical Lundquist number for instability $S = LV_A/\eta > S_c$
- Each current sheet segment undergoes a hierarchy of instabilities until it is marginally stable, i.e., $L_c = S_c \eta / V_A$
- Assuming each step of the hierarchy is Sweet-Parker-like, the thickness is $\delta_c = L_c/\sqrt{S_c} = \sqrt{S_c}\,\eta/V_A$
- Reconnection rate, independent of resistivity ~ $V_A/\sqrt{S_c}$
- Number of plasmoids scales as $L/L_c \sim S/S_c$


Linear
Nonlinear
~S^3/8
~S
No. of plasmoids in [−0.25,0.25]
S
10^2
10^1
10^0
10^5
10^6
10^7

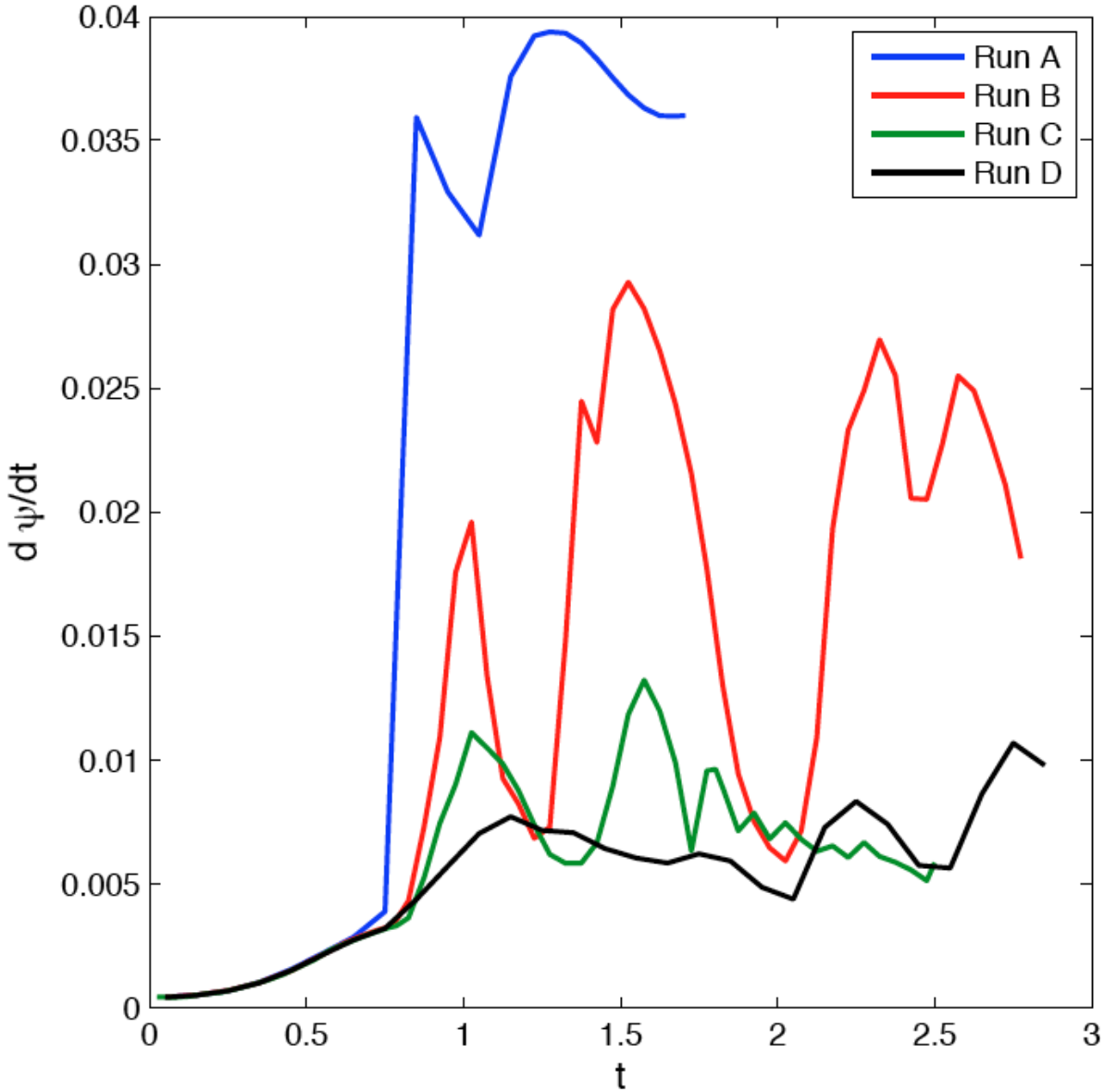
Parameter Space



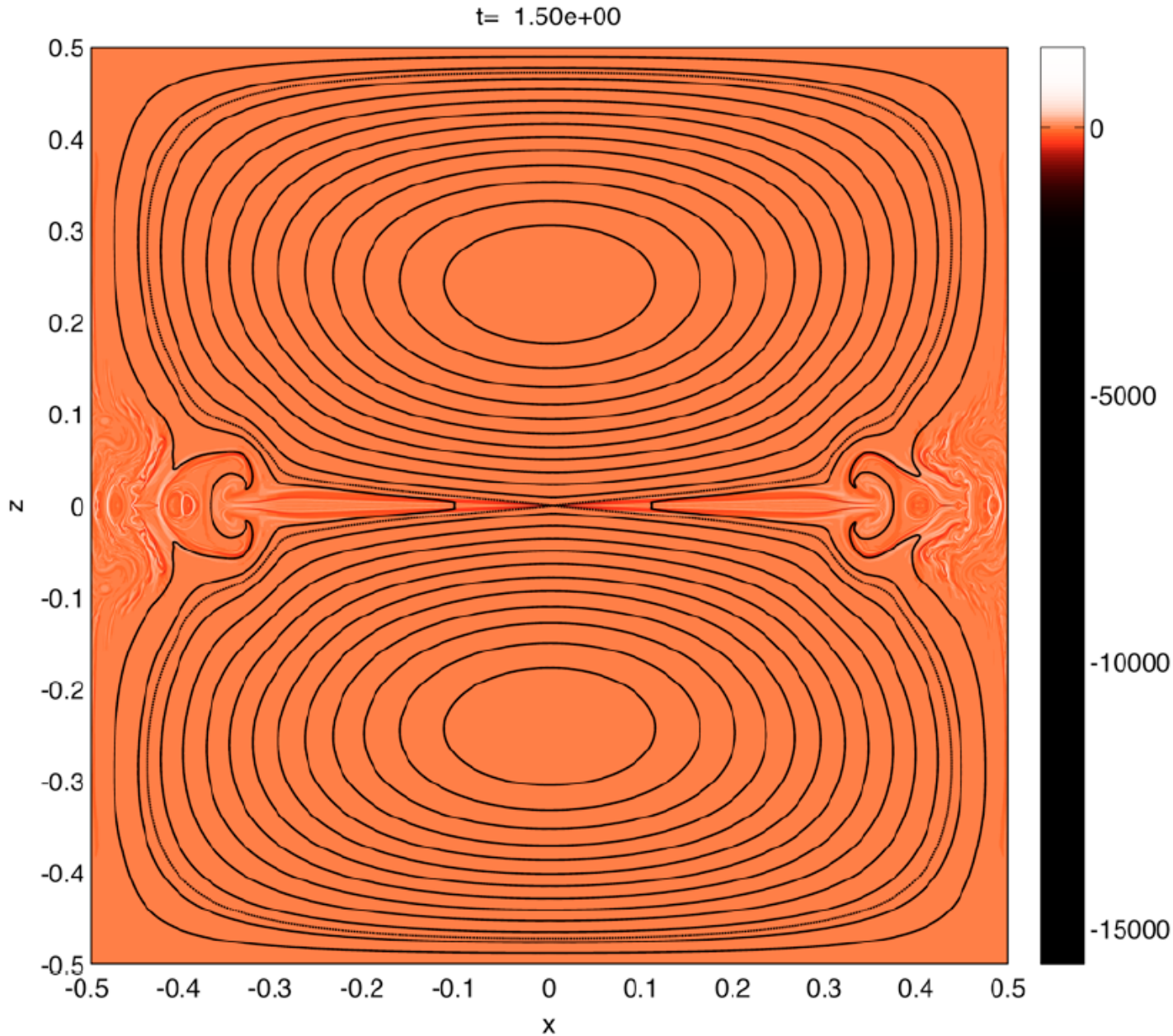
A: $S = 5 \times 10^5, d_i = 4 \times 10^{-4}$
B: $S = 5 \times 10^5, d_i = 2 \times 10^{-4}$
C: $S = 5 \times 10^5, d_i = 10^{-4}$
D: $S = 5 \times 10^5, d_i = 0$

Qualitatively similar results by Ji and Daughton (2012)

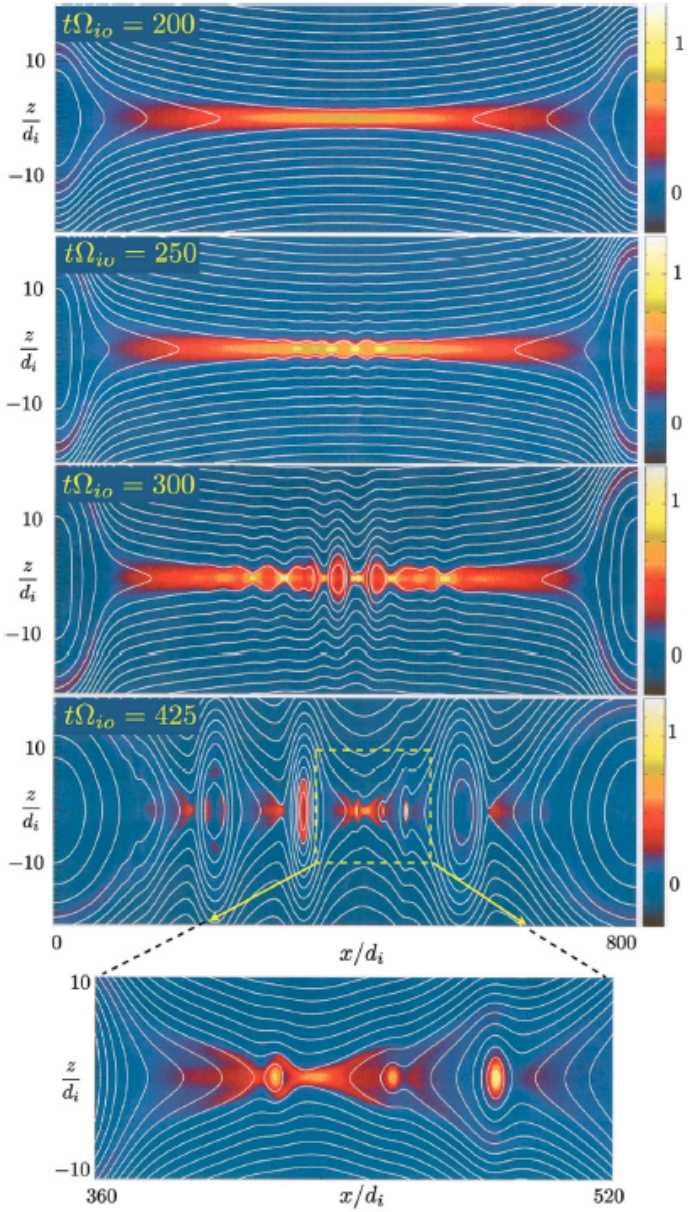
Reconnection Rate



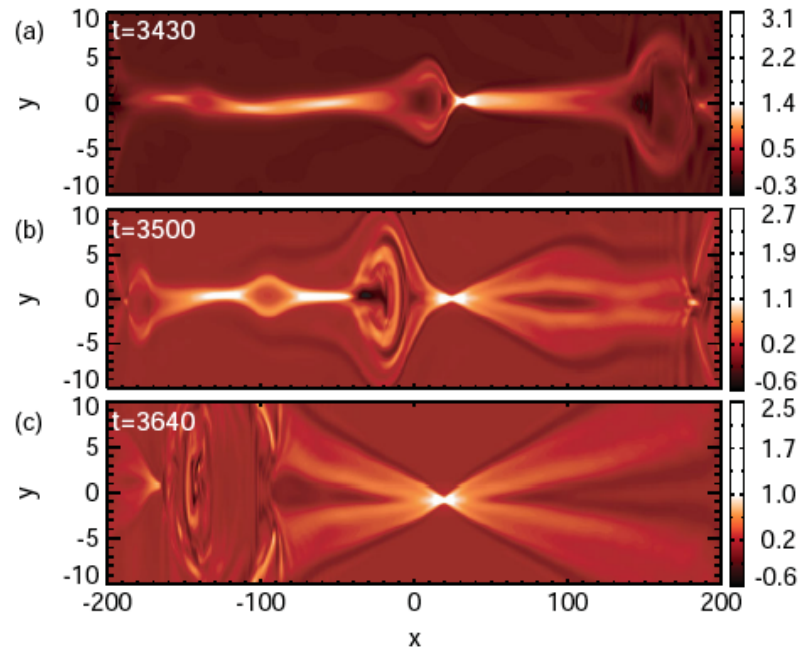
Run A, global configuration at late time



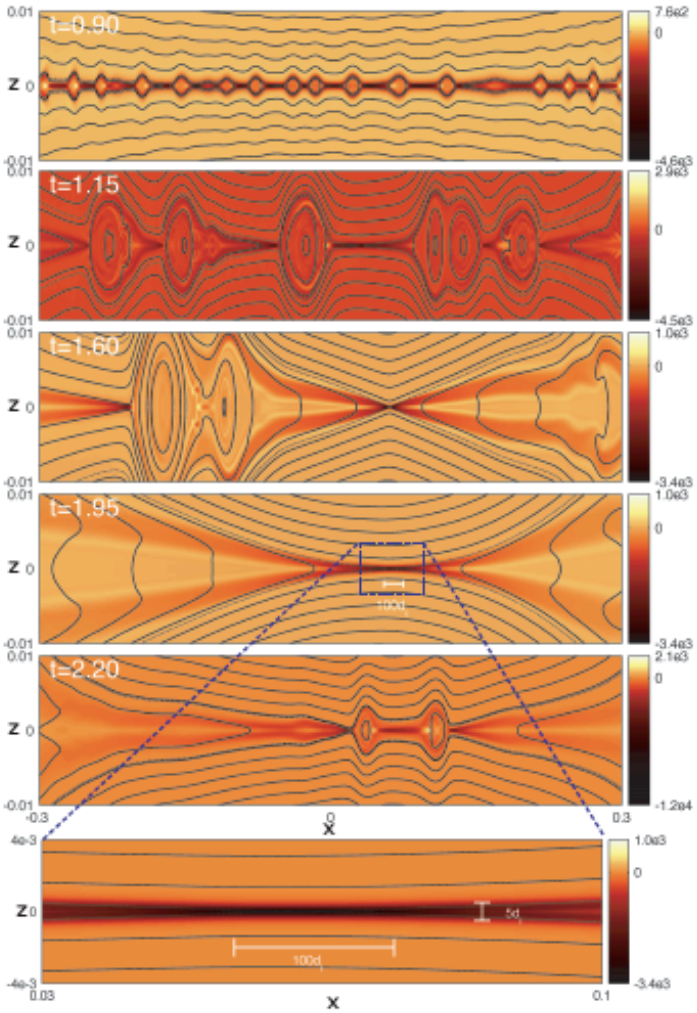
PIC and Hall MHD Simulations are Qualitatively Different



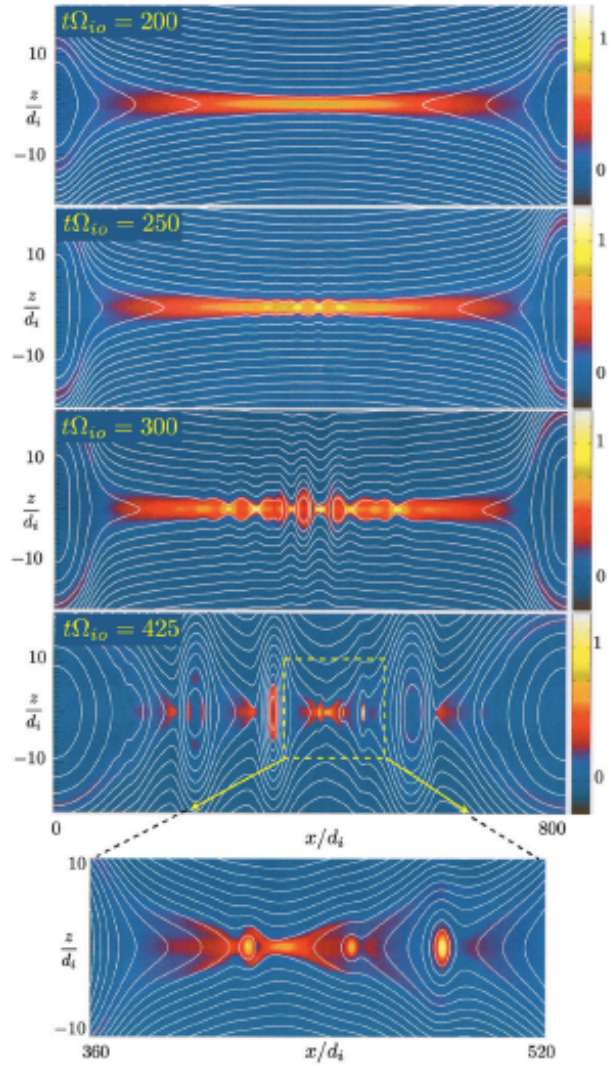
Daughton et al. (2009), PIC



Shepherd and Cassak (2010)
Resistive Hall MHD



Run B, resistive Hall

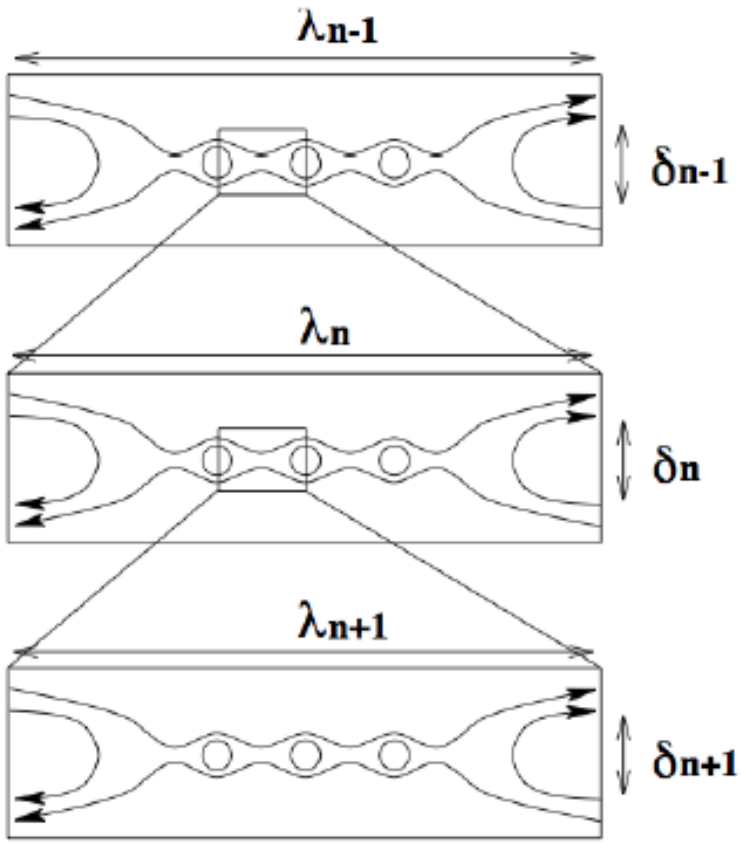


Daughton et al. (2009), PIC

Largest 2D Hall MHD simulation to date

Distribution of the Plasmoids

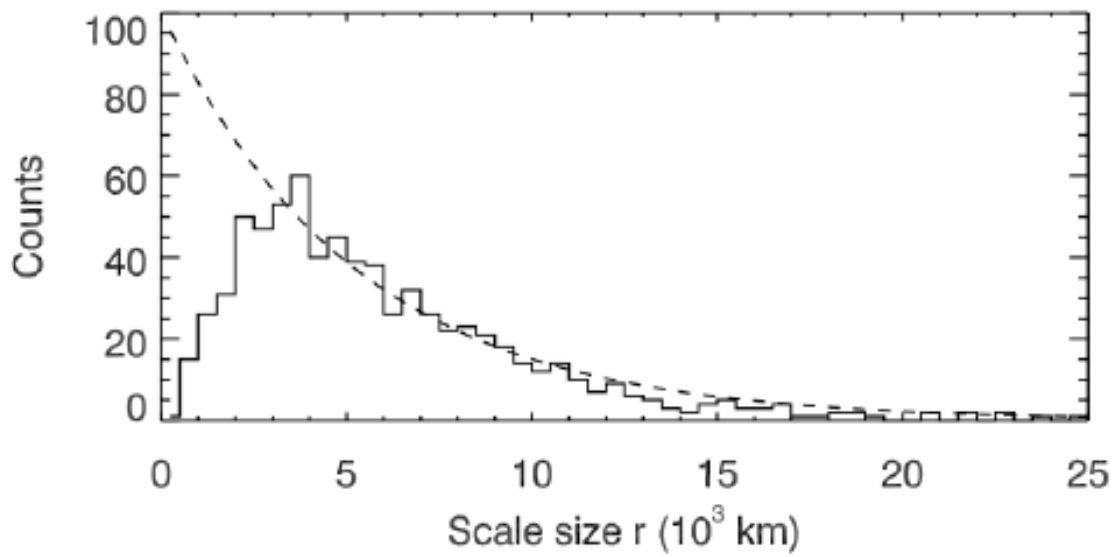
- Multiple levels of cascade suggest self-similarity across different scales, which often leads to power-laws.
- Uzdensky *et al.* (2010) gave a heuristic argument that predicts $f(\psi) \sim \psi^{-2}$



Shibata & Tanuma (2001)

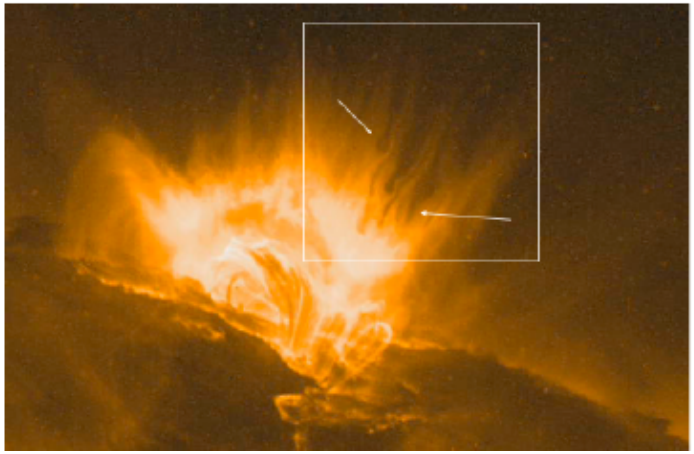
Distribution of the Plasmoids

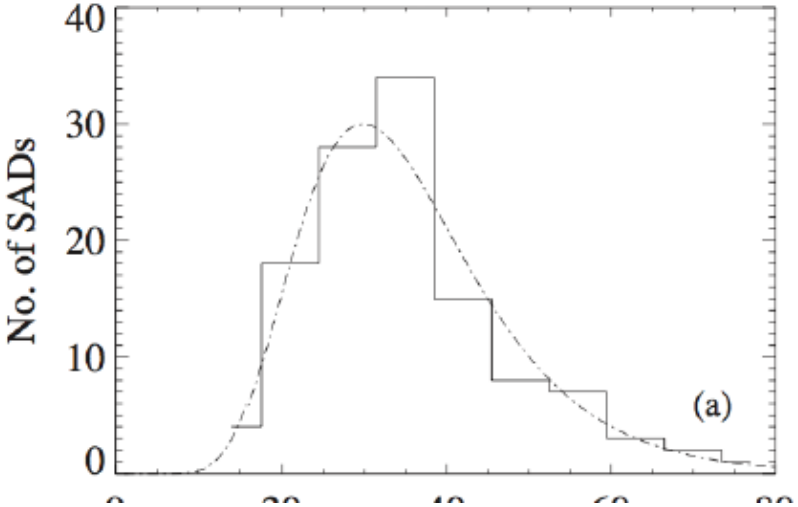
- Fermo *et al.* (2010) propose a statistical model that predicts $f(\psi)$ to decay exponentially in large ψ.
- The exponential tail is confirmed by observation data of flux transfer events (FTEs) in magnetopause by Cluster (Fermo *et al.* 2011).



Distribution of the Plasmoids

- McKenzie and Salvage (2011) claimed the sizes of supra-arcade downflows (SADs) follow a log-normal distribution.





Governing Equation for Plasmoid Distribution

Let $f(\psi)$ be the distribution function, $N(\psi) \equiv \int_\psi^\infty f(\psi')d\psi'$, $\tau_A \equiv L/V_A$. The governing equation can be written as

$$\frac{\partial f}{\partial t} + \gamma \frac{\partial f}{\partial \psi} = \zeta\delta(\psi) - \frac{fN}{\tau_A} - \frac{f}{\tau_A}$$

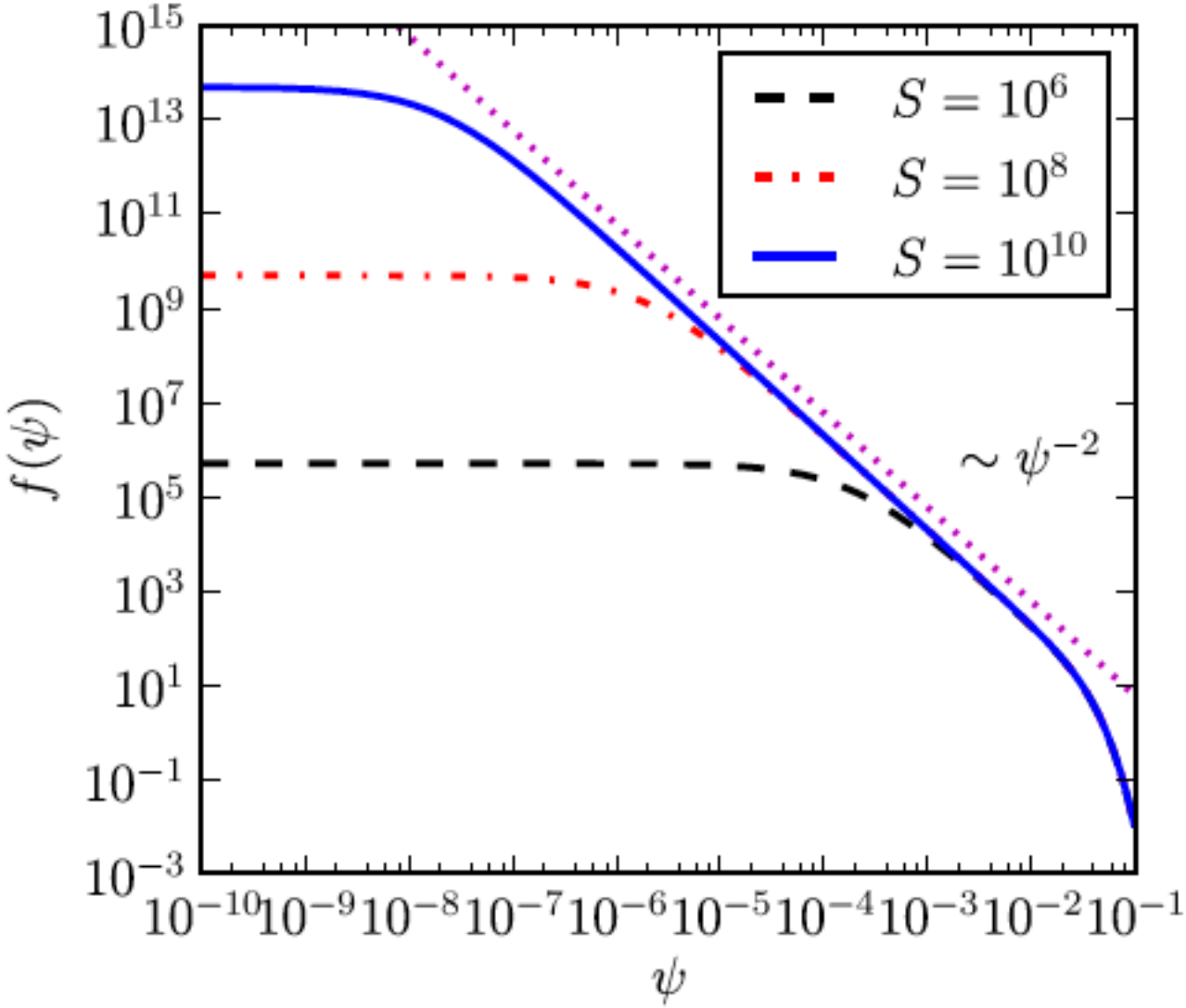
- All plasmoids grow at the same rate $\gamma \sim BV_A/\sqrt{S_c}$
- When new plasmoids are generated, they contain zero flux — represented by the source term $\zeta\delta(\psi)$
- Plasmoid loss due to coalescence with larger plasmoids $-fN/\tau_A$ — assuming the relative speed between plasmoids $\sim V_A$
- Plasmoid loss due to advection $-f/\tau_A$

Steady State Solution of The Governing Eq.

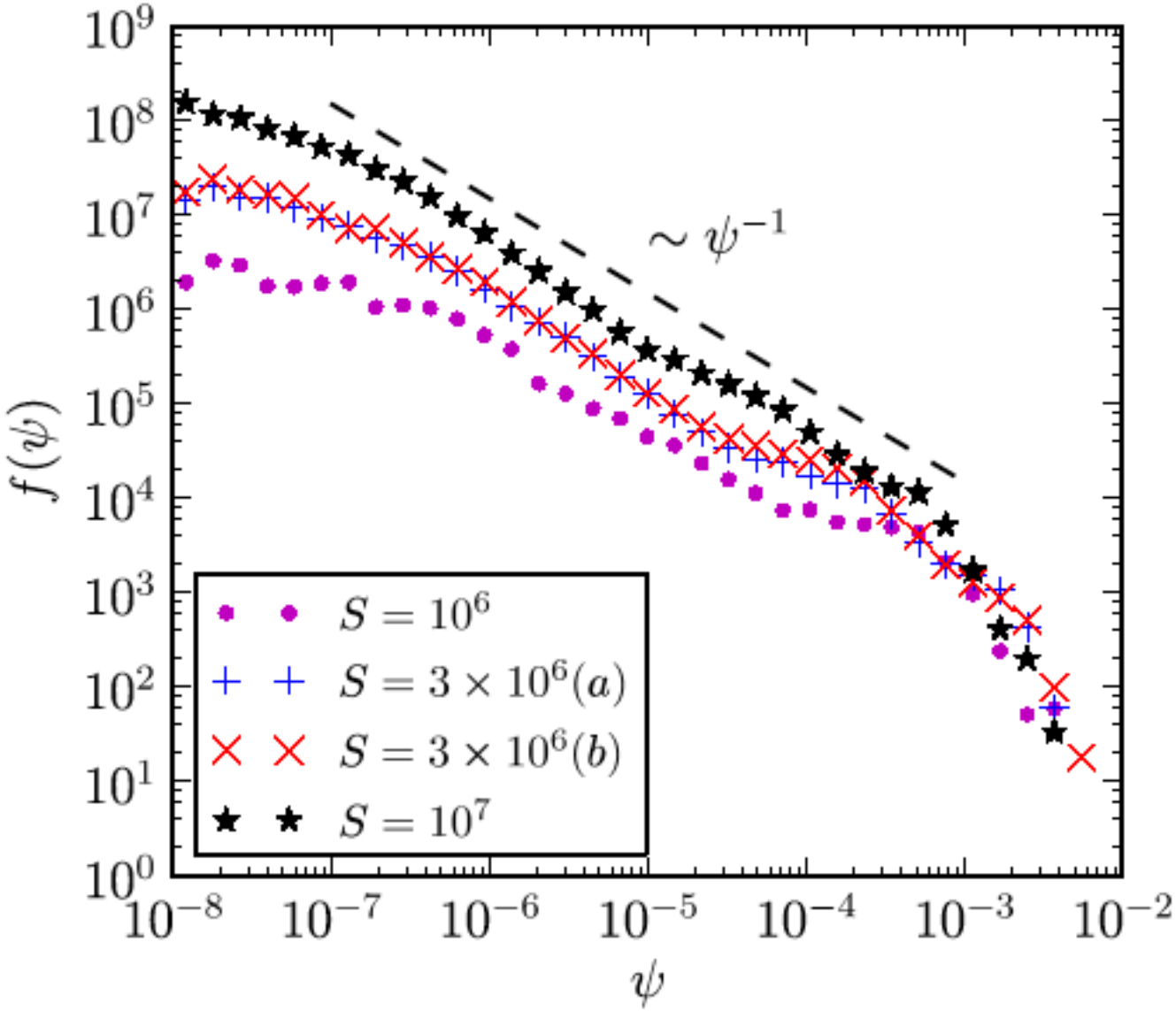
$$f(\psi) = \frac{2C/\gamma\tau_A}{\left(C - \exp(-\psi/\gamma\tau_A)\right)^2} \exp(-\psi/\gamma\tau_A)$$

- $C = 1 + 2/N_0$
- $\zeta\tau_A = N_0^2/2 + N_0 \simeq N_0^2/2$ in high S limit.
- Three distinct regimes in the high-S limit :
  - $f \simeq (2/\gamma\tau_A)\exp(-\psi/\gamma\tau_A)$ when $\psi/\gamma\tau_A \gg 1$— advection is the dominant loss mechanism
  - $f \simeq 2\gamma\tau_A\psi^{-2}$ when $2/N_0 \ll \psi/\gamma\tau_A \ll 1$ — coalescence is the dominant loss mechanism
  - $f \simeq N_0^2/2\gamma\tau_A$ when $\psi/\gamma\tau_A \ll 2/N_0$

Steady State Solution of The Governing Eq.



Plasmoid Distribution from Simulations



Relative Speed Decreases with Increasing ψ

- The plasmoid-dominated reconnection layer organizes itself into a state such that large plasmoids tend to avoid merging with each other.
- Why is that?
  - The flux of a plasmoid is approximately proportional to its age. Large plasmoids are old plasmoids.
  - A plasmoid can become large only if it has not encountered plasmoids larger than itself for an extended period of time.
  - Plasmoids with high speeds relative to their neighboring plasmoids will encounter larger plasmoids and disappear easily, whereas those with small relative speeds are more likely to live for a long time and become large.

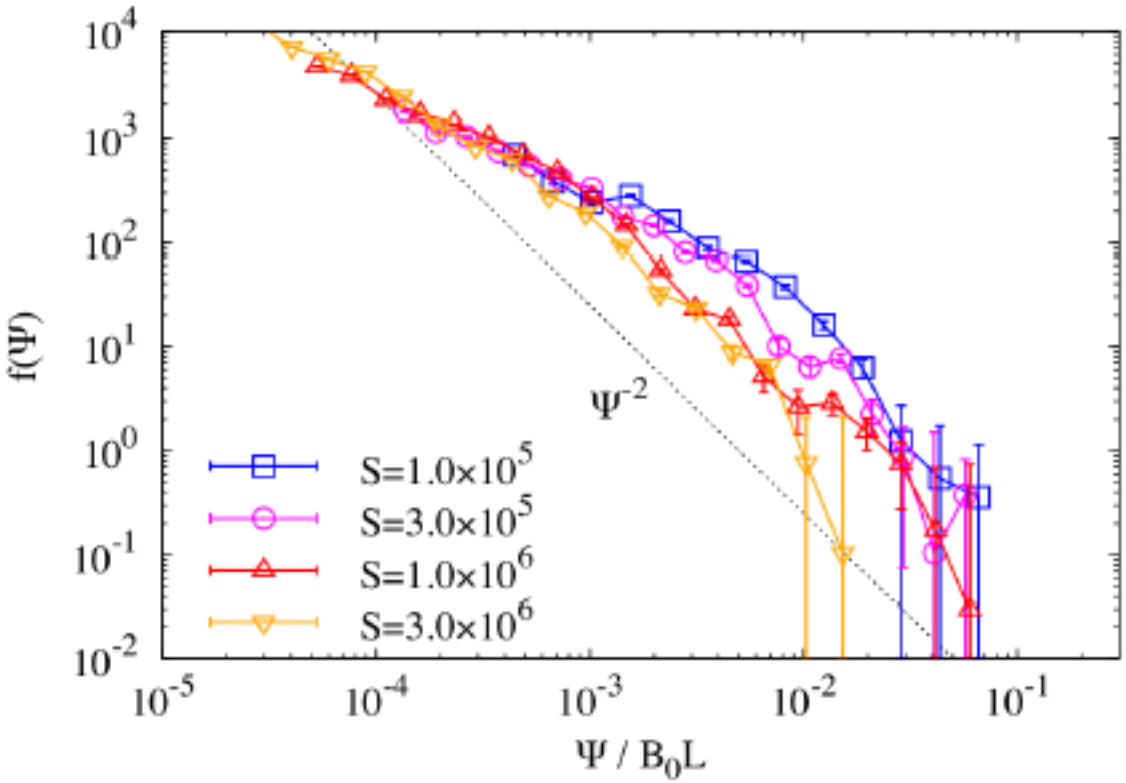
Modified Governing Equation

Consider a distribution function $F(\psi, v)$ of flux and relative speed. Let $f(\psi) = \int_0^\infty F(\psi, v)dv$ and $N(\psi) = \int_\psi^\infty f(\psi)d\psi$. We modify the governing Eq. as

$$\partial_t F + \gamma \frac{\partial F}{\partial \psi} = \zeta \delta(\psi) h(v) - \frac{v}{V_A} \frac{FN}{\tau_A} - \frac{F}{\tau_A}.$$

- $h(v)$ is a distribution function in relative speed when new plasmoids are generated, which is taken as arbitrary at this point.
- Plasmoid loss term due to merging is proportional to v.

Distribution from Loureiro et al. arXiv:1108.4040



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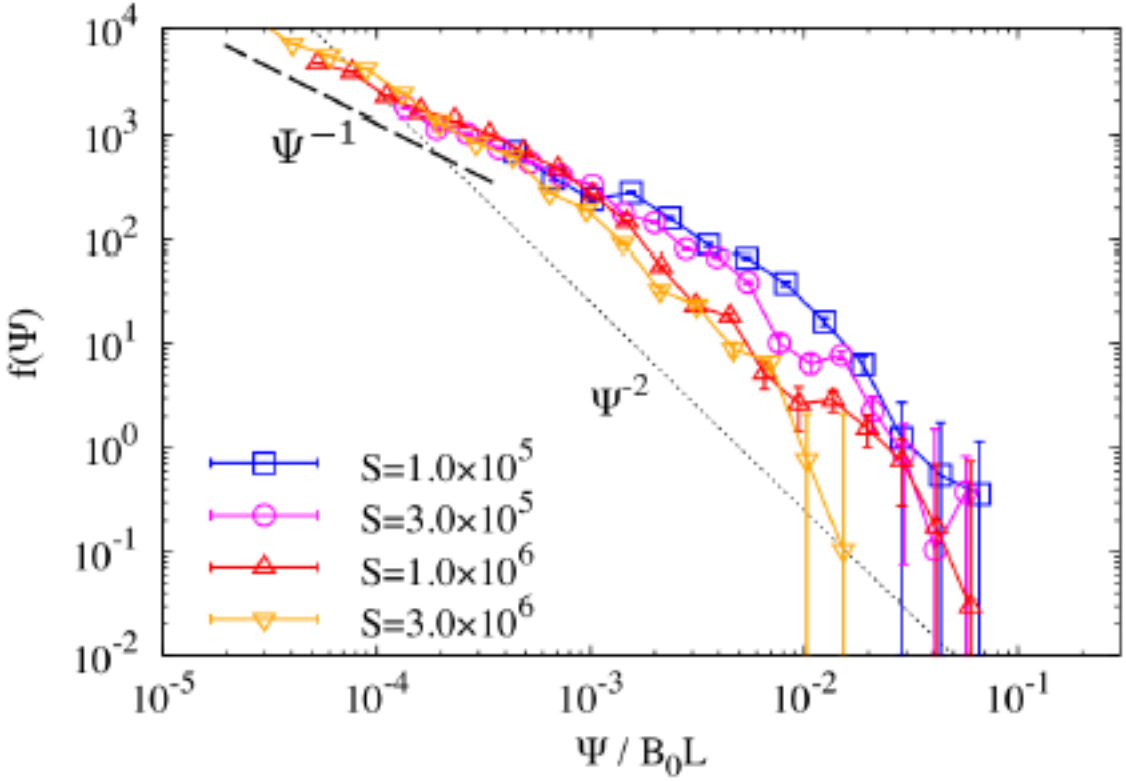


Summary

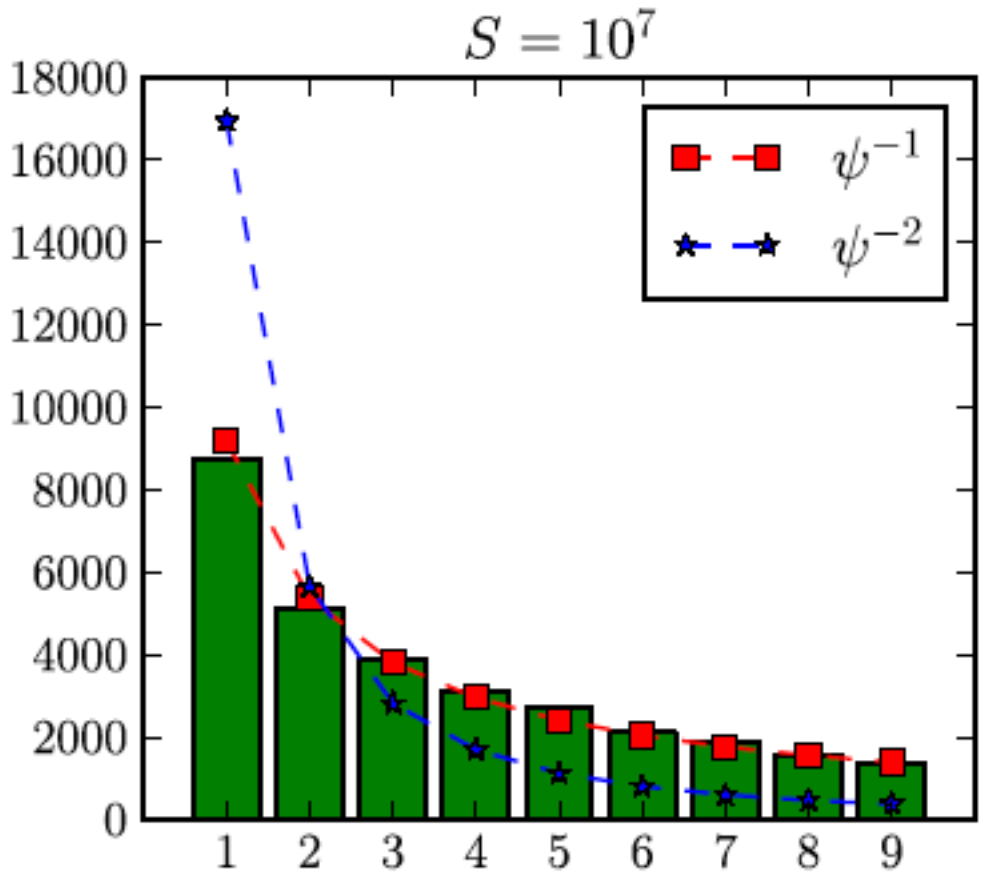
Onset of fast reconnection, mediated by the dynamics of thin current sheets, in high-Lundquist-number laboratory and space plasmas. Two mechanisms:

- Hall MHD, seen in theory and laboratory experiments of *moderate size*, when the Sweet-Parker width falls below the ion skin depth.
- Fast plasmoid instability of thin current sheets in *large systems*, substantially exceeding Sweet-Parker rates within the realm of resistive MHD, without invoking Hall current and/or electron pressure tensor effects, or 2D/3D turbulence (Matthaeus and Lamkin 1986, Loureiro et al. 2009, Lazarian and Vishniac 1999, Kowal et al. 2009). The peak reconnection rate ~ $0.01V_A$.

Summary (continued)

- Hall MHD effects can enhance the peak reconnection rate an order of magnitude higher ~0 .1V_A.
- Hall MHD does not inevitably produce a single X-point. In an intermediate regime of fast reconnection, the system can alternate between a single X-point state and a state with multiple X-points produced by the plasmoid instability.
- In geometries such as the tokamak or the Earth's magnetotail, secondary ballooning instabilities can also halt or quench reconnecting instabilities. In these cases, the further evolution of the system may be controlled by ideal instabilities.
- 3D evolution of such systems is a subject of great future interest.

The Distribution of Leading Digit of Flux Follows Benford's Law



- If $f \sim \psi^{-1}$: Benford's Law $P(d) = \log_{10}(1 + 1/d)$
- If $f \sim \psi^{-2}$: $P(d) = 10/9d(d + 1)$.