

Radiative MHD Simulation of Relativistic Magnetic Reconnection

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contents

1. Numerical results of the Relativistic MHD reconnection

The gas interacts only with the magnetic fields.

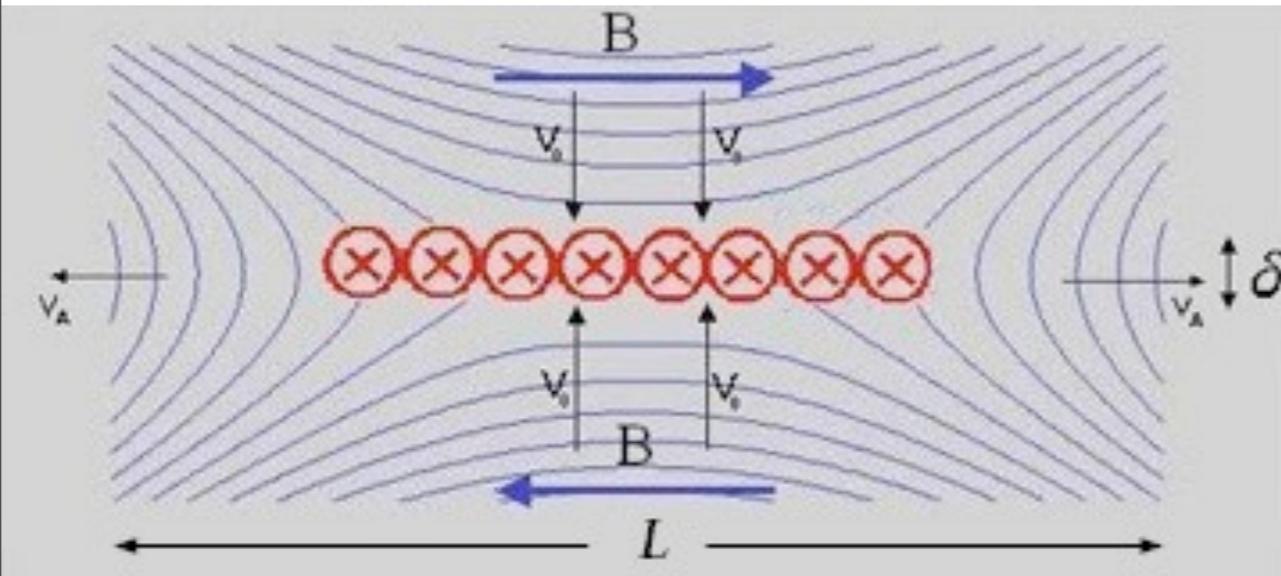
2. Numerical results of the Magnetic Reconnection in Uniform Radiation Field

The gas interacts with the magnetic fields and the radiation fields.

Non-relativistic MHD reconnection model

Sweet-Parker model

Sweet '58, Parker '57



Magnetic energy is dissipated by Ohmic diffusion in the diffusion region.

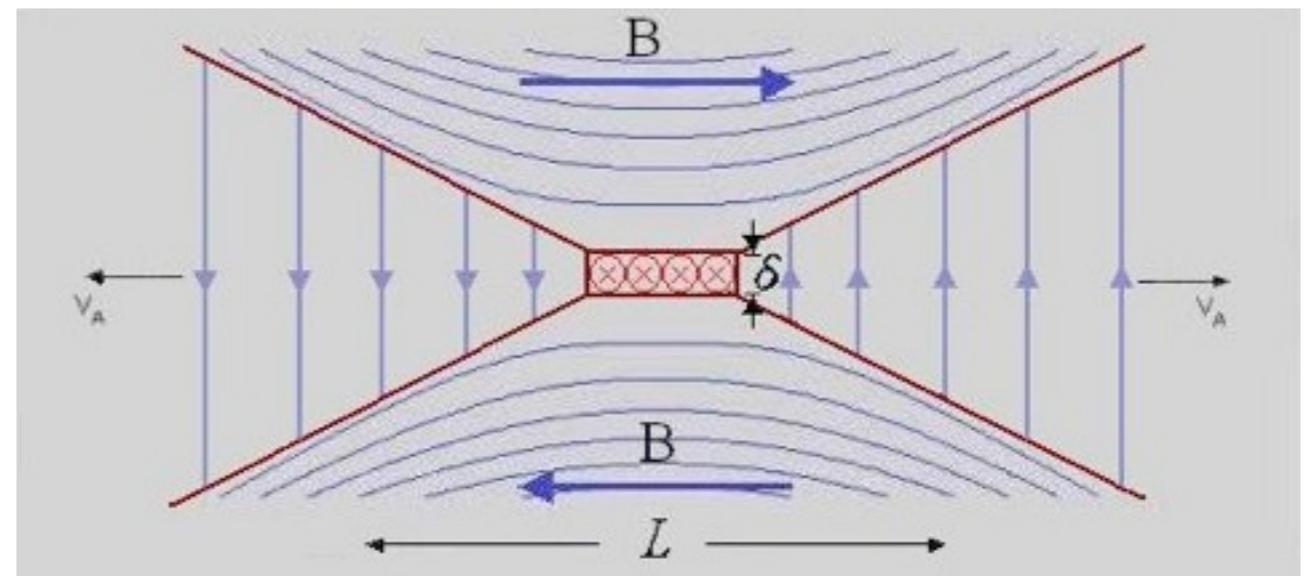
outflow speed ~ Alfvén vel.

reconnection rate $\mathcal{R} \simeq R_M^{-0.5}$

slow reconnection rate

Petschek model

Petschek '64



Magnetic energy is liberated not only the diffusion region but mainly at the slow shock.

outflow speed ~ Alfvén vel.

reconnection rate $\mathcal{R} \simeq (\log R_M)^{-1}$

faster energy conversion

<http://www.psfc.mit.edu>

Outflow velocity and reconnection rate

◆ mass conservation relation between the inflow and outflow:

$$\rho_i v_i \gamma_i L = \rho_o v_o \gamma_o \delta$$

→ $\frac{v_i}{v_o} = \frac{\rho_o \delta \gamma_o}{\rho_i L \gamma_i} \quad (1) \quad \text{where } \gamma = (1 - v^2/c^2)^{-1/2}$

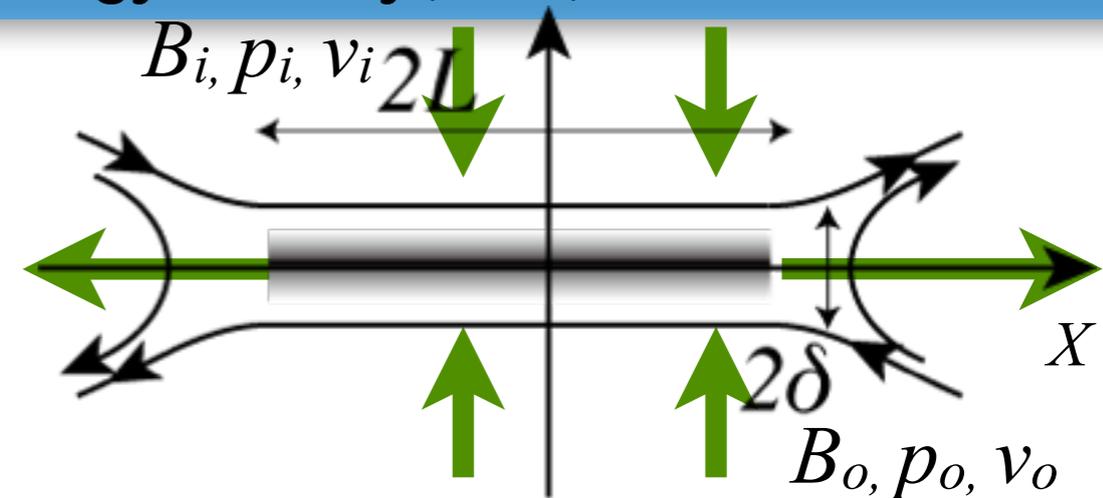
= (compression ratio)

x (aspect ratio)

x (Lorentz contraction)

Relativistic means...

that the magnetic energy density ($B^2/8\pi$) exceeds the rest mass energy density (ρc^2).



◆ **non-relativistic** ($\rho c^2 \gg B^2/8\pi$)

Magnetic energy is converted into the kinetic energy:

$$\rightarrow \frac{1}{2} \rho v_o^2 \sim \frac{B^2}{8\pi} \rightarrow v_o \simeq V_A = \frac{B}{\sqrt{4\pi\rho}} \quad (2)$$

→ $\frac{v_i}{v_o} \simeq \frac{v_i}{V_A} \simeq \begin{cases} R_M^{-0.5} & \text{Sweet-Parker} \\ (\log R_M)^{-1} & \text{Petscheck} \end{cases}$

See, also
Blackman & Field '94
Lyutikov & Uzdensky '03
Lyubarsky '05

◆ **relativistic** ($\rho c^2 \ll B^2/8\pi$)

→ from equation 2, outflow velocity approaches to c for a larger B:

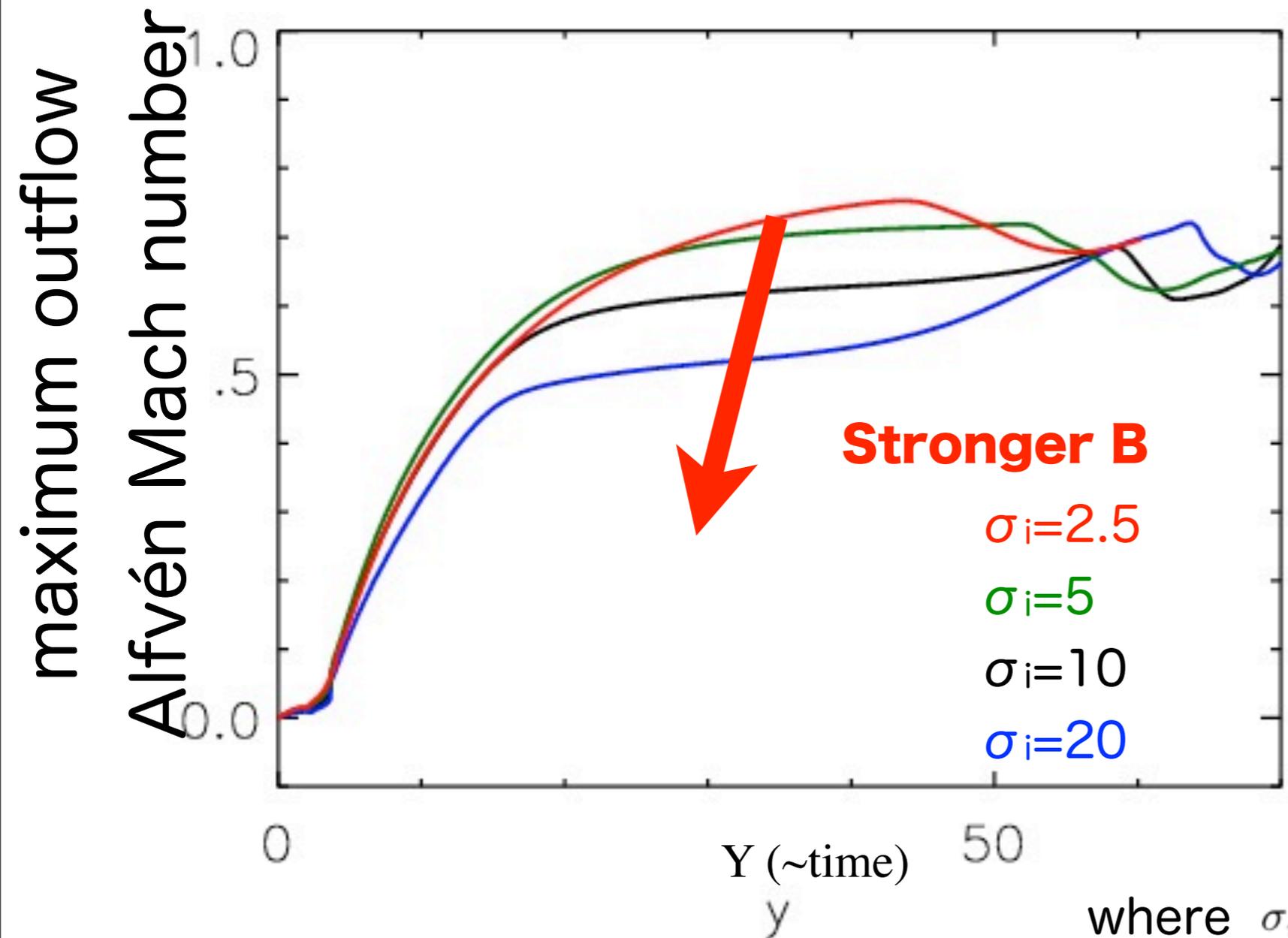
→ from equation 1, rec. rate might be enhanced by factor γ_o in relativistic regime?

Relativistic Sweet-Parker MRX

Relativistic Resistive Magnetohydrodynamic (R2MHD) simulations

Maximum outflow velocity on the plane of $x=0$.

Takahashi+ 11



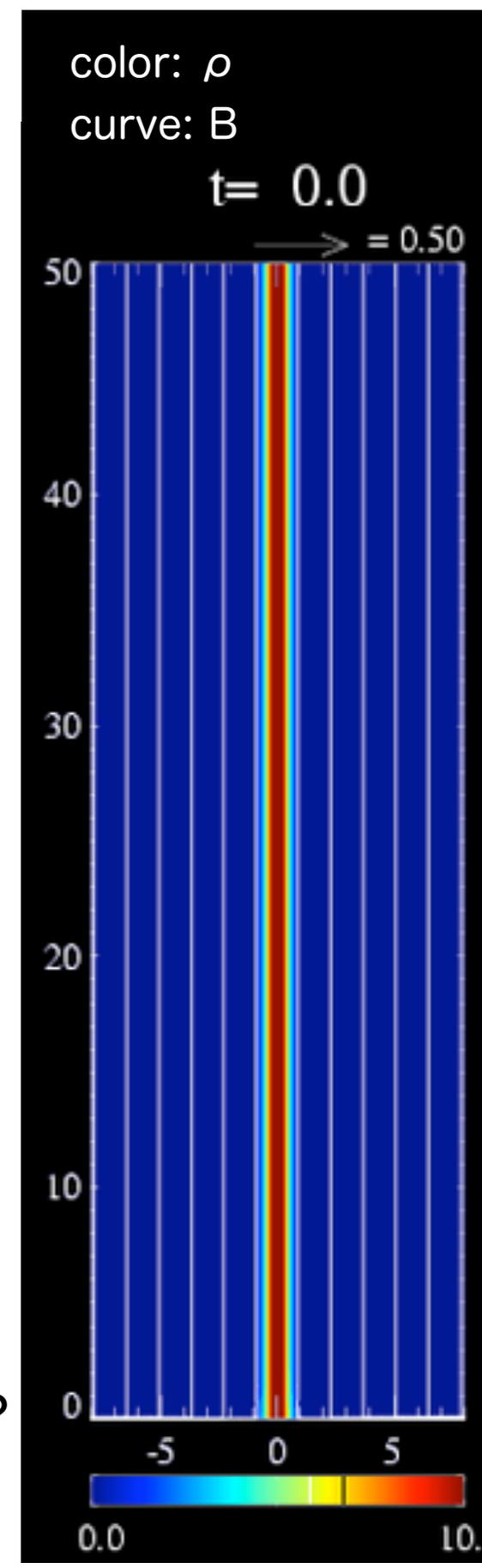
where $\sigma_i = \frac{B_i^2}{4\pi\rho_i c^2}$

M_A increases with time and it saturates.

The saturated M_A is smaller for a larger B in the relativistic regime?

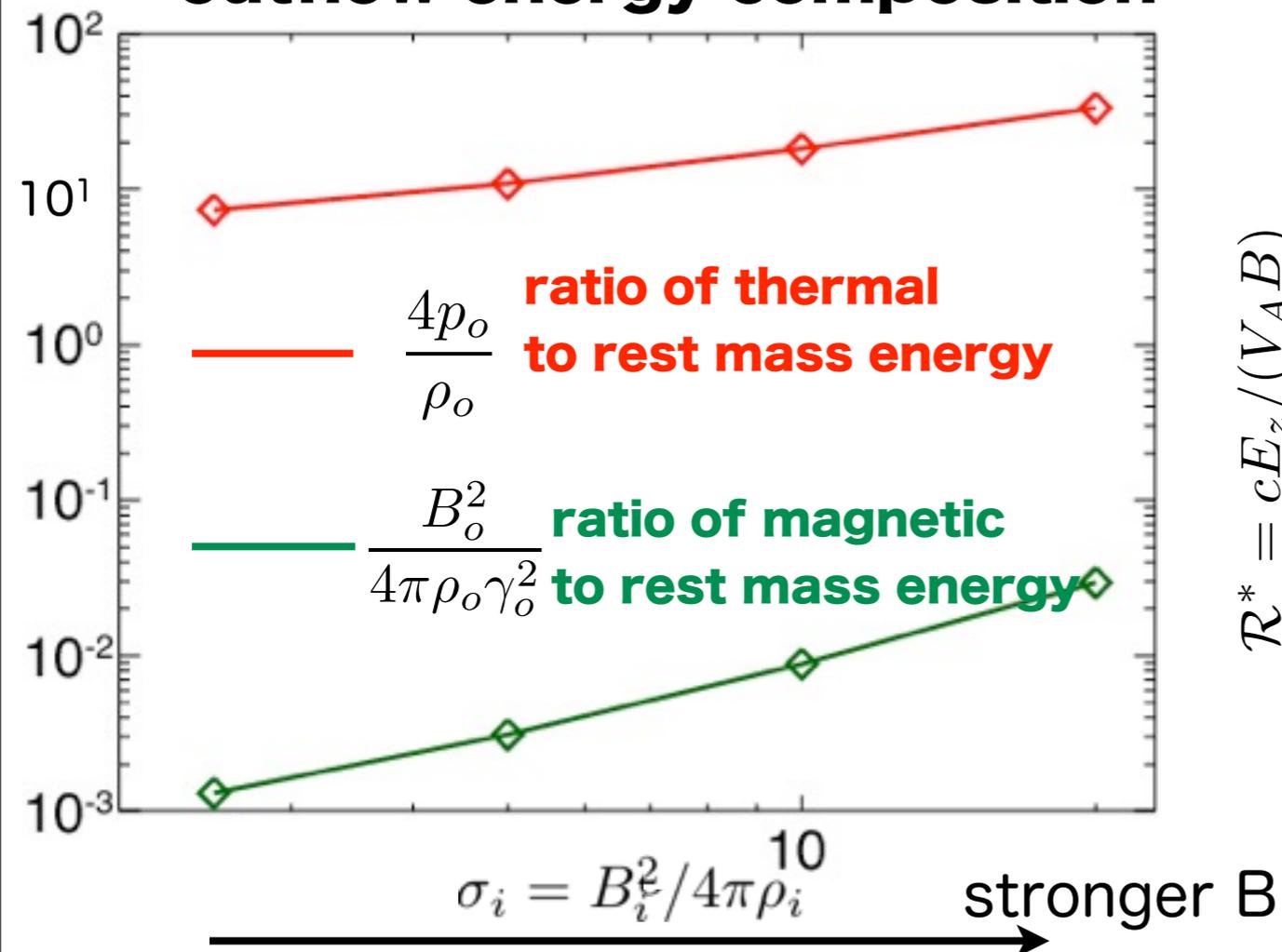
Mach number seems to decrease with increasing B

mildly relativistic outflow

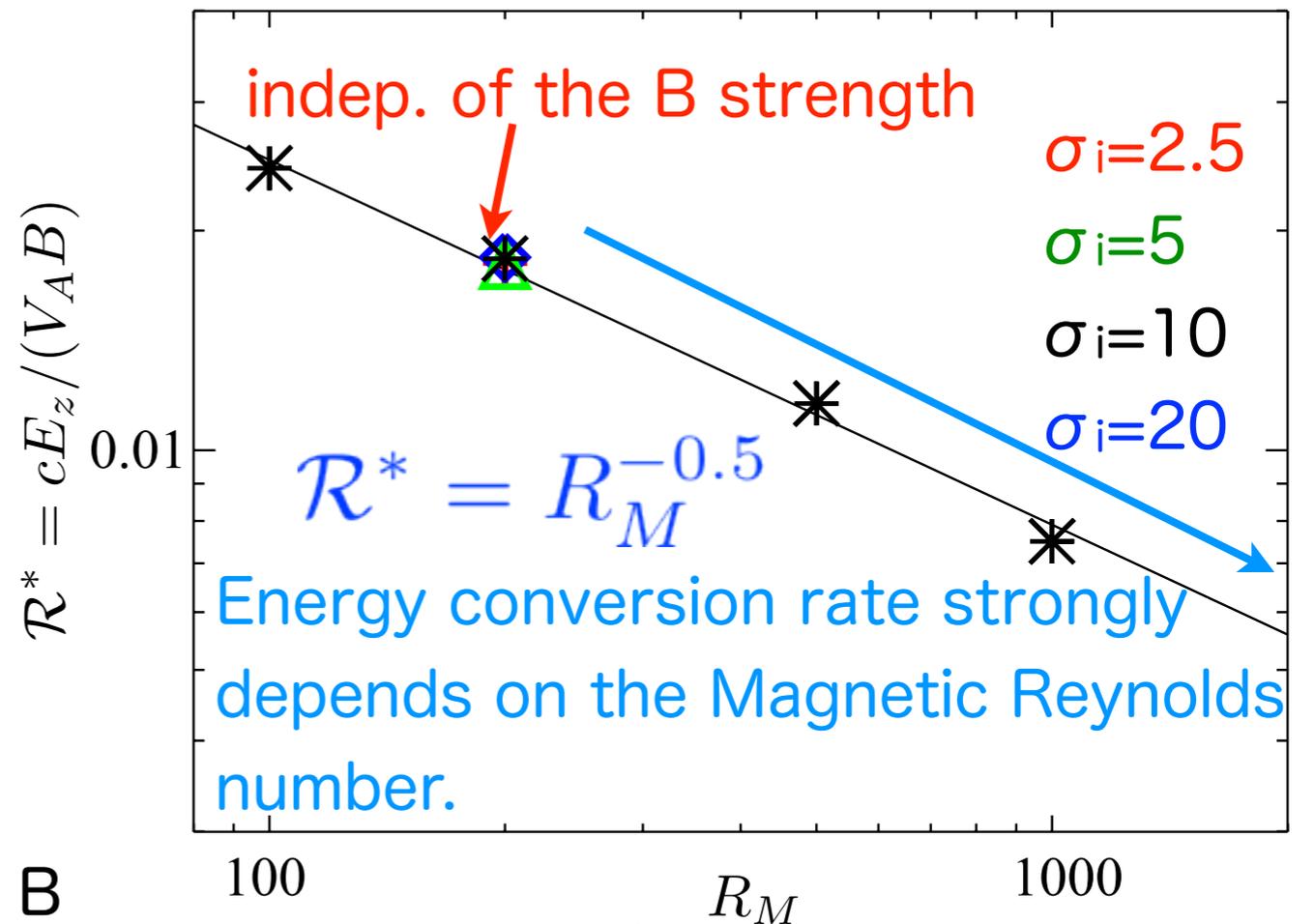


Reconnection rate

outflow energy composition



reconnection rate



Most of the magnetic energy is converted into the thermal energy.

-> It increases the plasma inertia (since $E=mc^2$) (Lyubarsky '05).

-> Plasma cannot be accelerated up to the relativistic velocity due to the inertia.

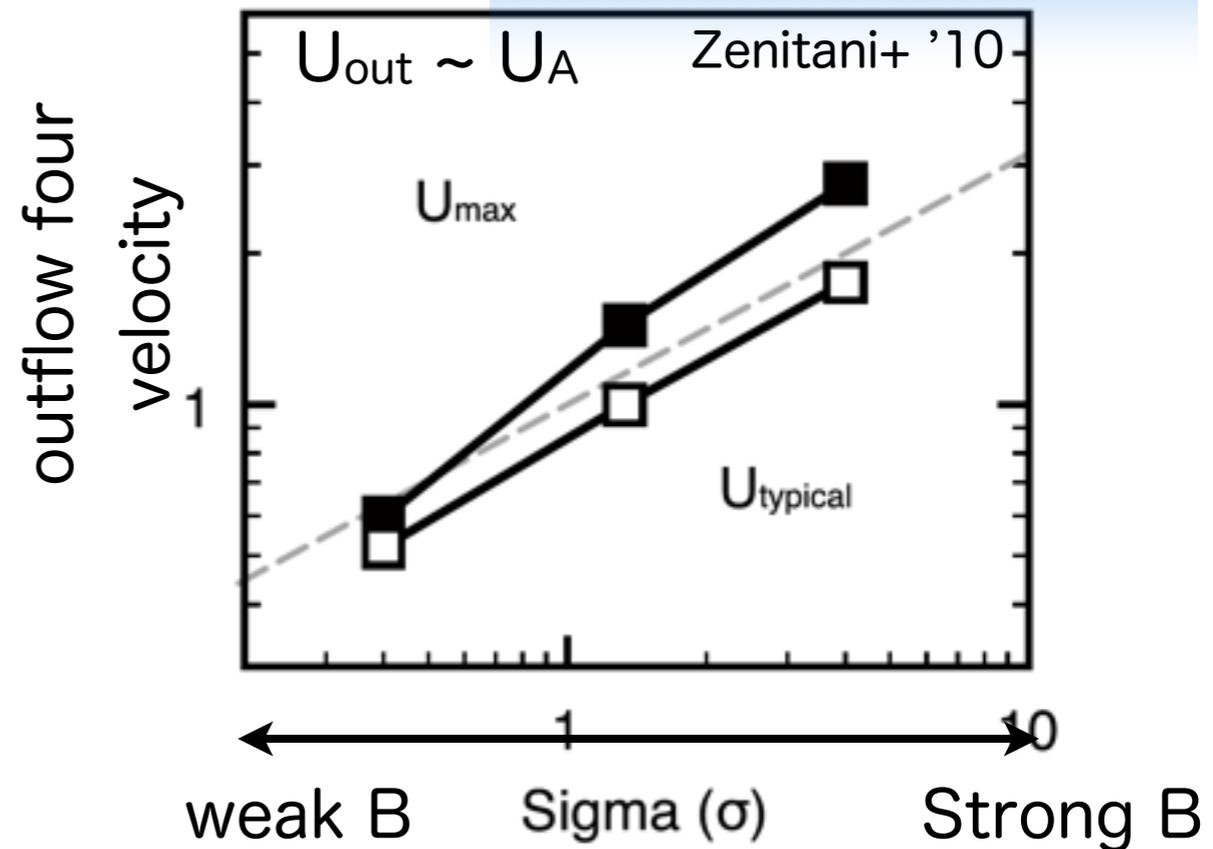
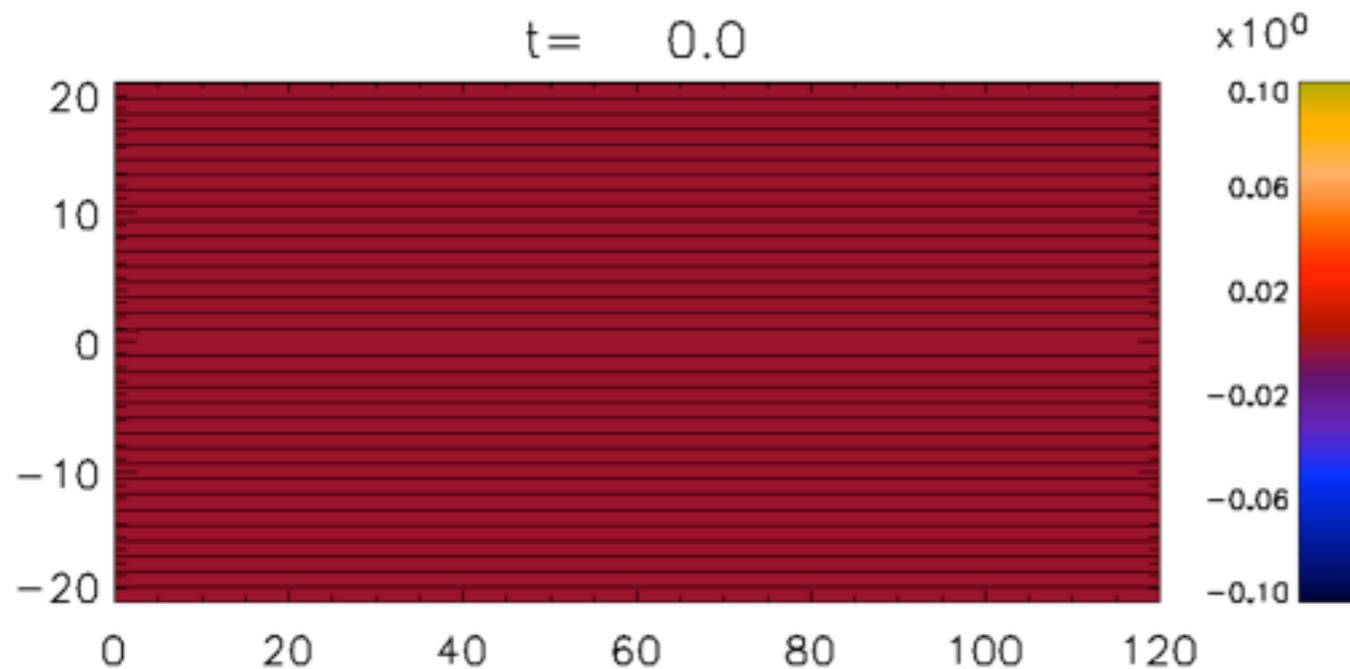
-> Relativistic effects cannot be expected in the reconnection rate ($r \sim 1$).

-> SP reconnection is the slow energy conversion process

Relativistic Petschek type MRX

Numerical simulations of the Relativistic Petschek type magnetic reconnection with spatially localized resistivity.

see,
Watanabe & Yokoyama '06,
Zenitani et al. '10,
Zanotti & Dumbser '11



Relativistic Petschek type Magnetic Reconnection.

Outflow velocity is accelerated up to the Alfvén velocity by the magnetic tension f .

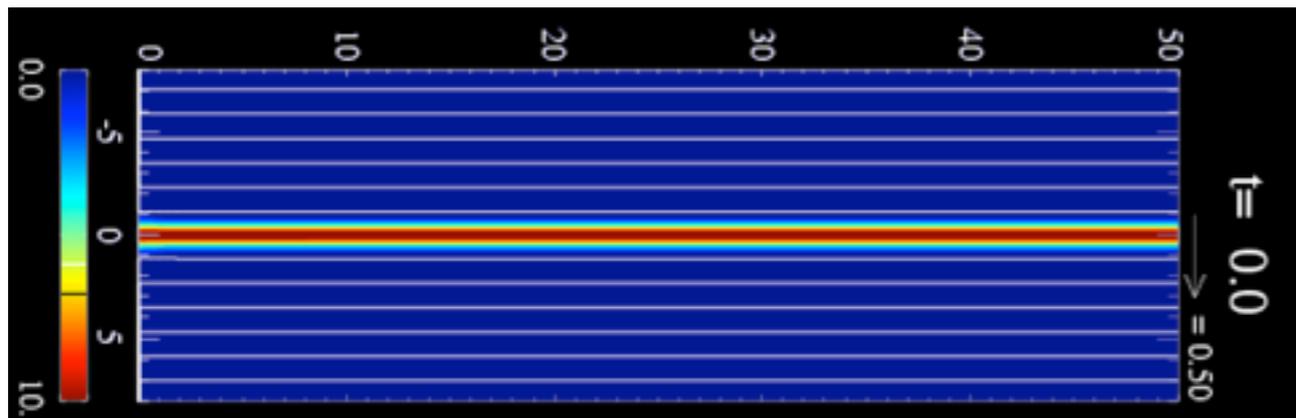
Reconnection rate is enhanced in the relativistic regime (Watanabe & Yokoyama '06).

The thermal energy is comparable to the kinetic energy in the outflow (Zenitani '09)

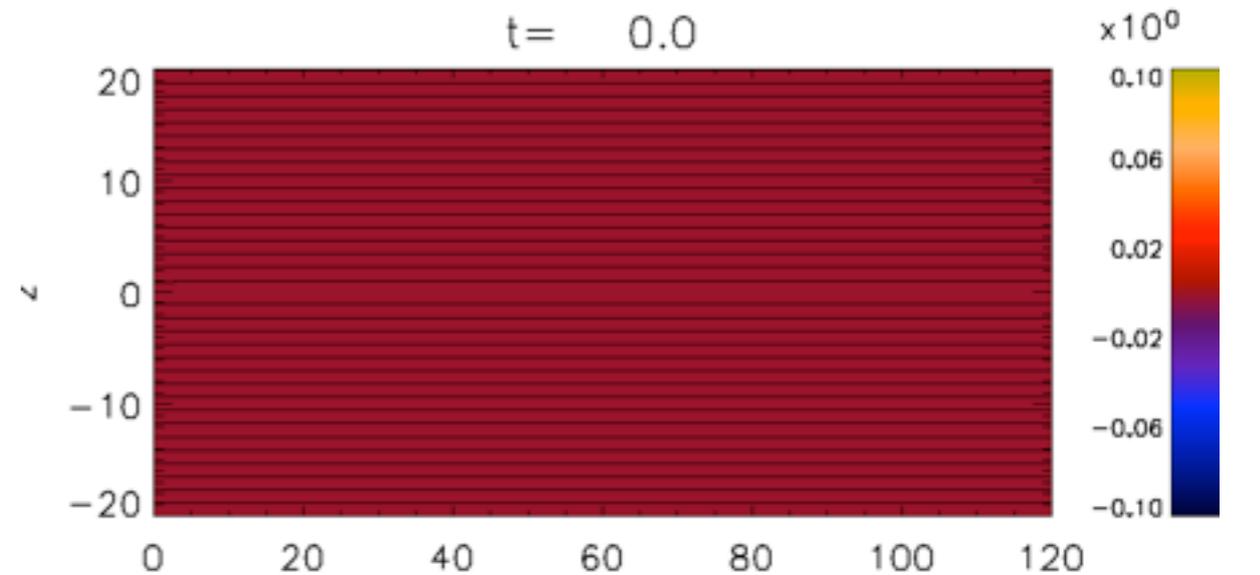
Summary of Relativistic Magnetic Reconnection

Sweet-Parker model

Petschek model



Takahashi+ '11



Watanabe & Yokoyama '06, Zenitani+ '10, Zanotti & Dumber '11

Magnetic energy is liberated by Ohmic dissipation in the diffusion region.

mildly relativistic outflow

reconnection rate $\mathcal{R} \simeq R_M^{-0.5}$

slow reconnection rate

Magnetic energy is liberated not only the diffusion region but mainly at the slow shock

outflow speed \sim Alfvén velocity \sim relativistic

reconnection rate $\mathcal{R} \simeq (\log R_M)^{-1}$

faster energy conversion

?

We observed the growth of tearing ins.

It is expected that the reconnection transitions from slow SP MRX to fast turbulent MRX

in relativistic regime. (see also Zanotti & Dumber '11)

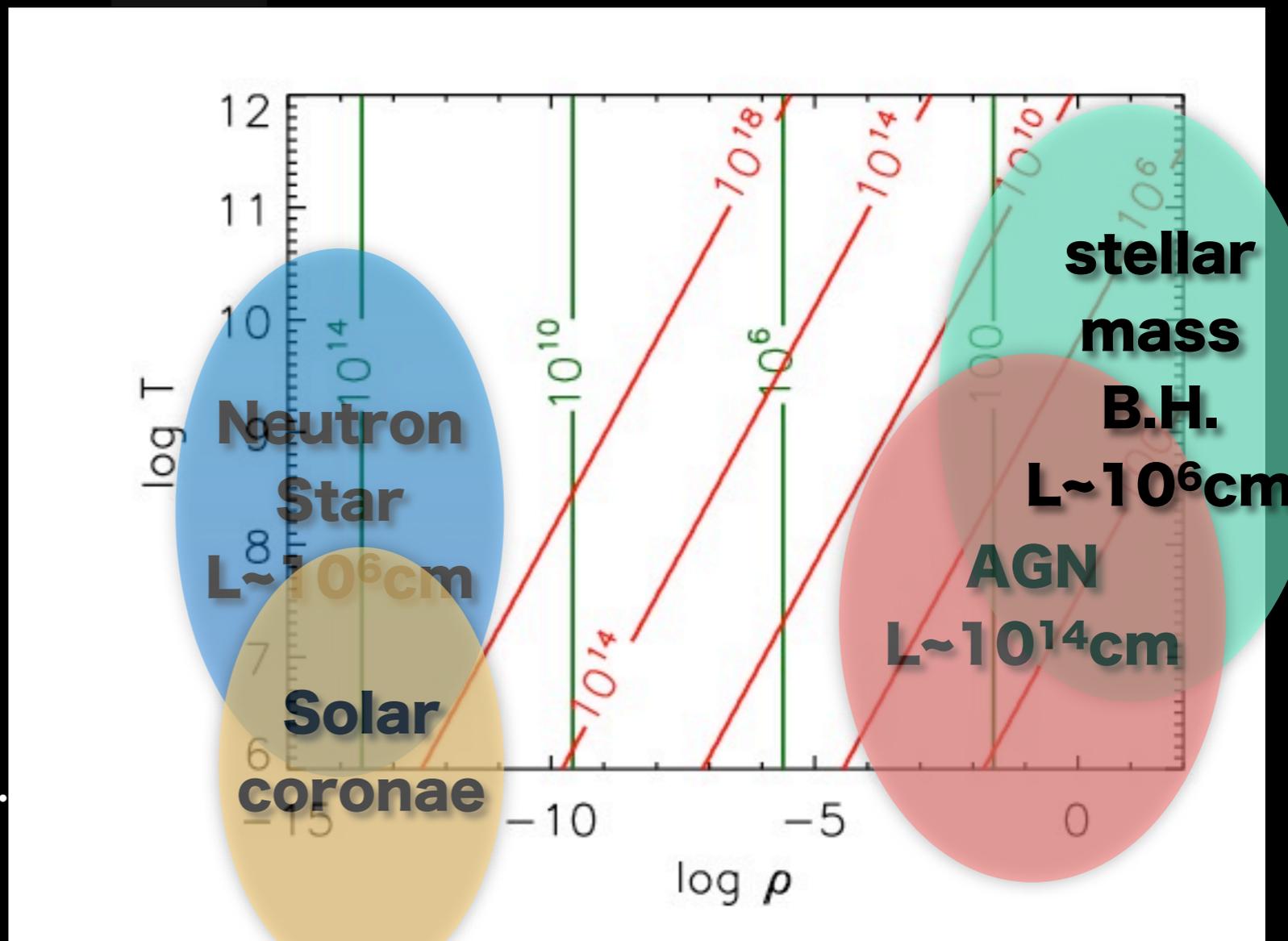
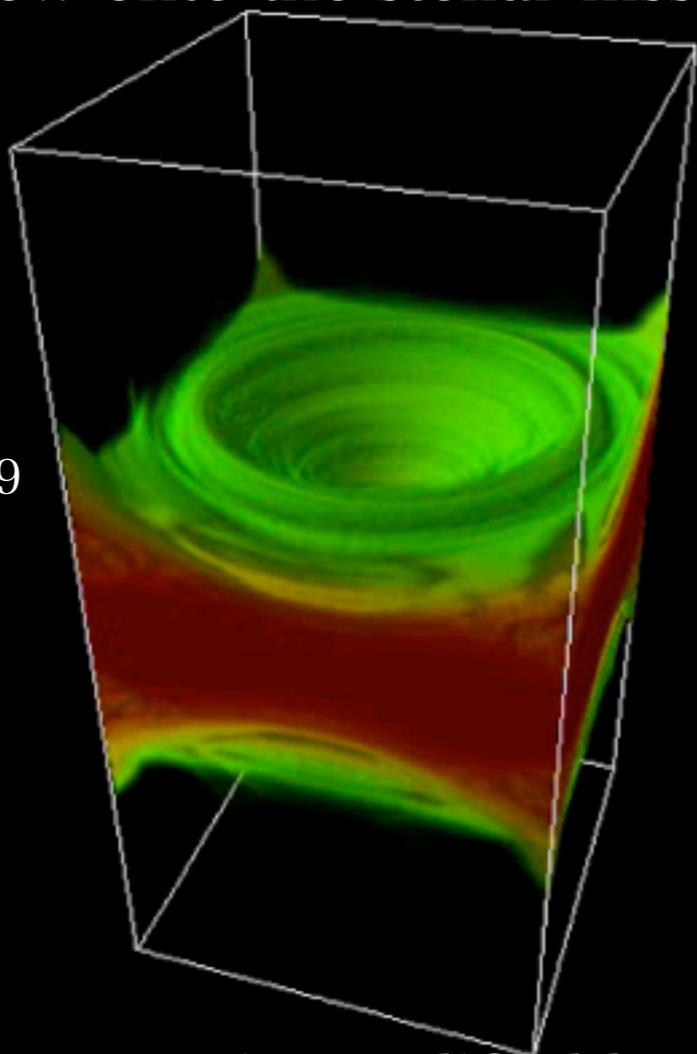
High energy astronomical environment...

Radiation effect cannot be ignored

non-relativistic radiation MHD
simulation of super critical accretion
flow onto the stellar mass BH.

mean free path for
electron scattering
free-free emission

Ohsuga '09



Magnetic energy is amplified by MRI.
Part of B energy is converted to
thermal energy through MRX in
accretion disks.

**We develop the Relativistic
Resistive Radiation MHD (R3MHD) code.**

How to treat the radiation field?

Radiation transfer equation:
7 indep. variables

$$\frac{\partial I}{\partial t} + \mathbf{n} \cdot \nabla I = \chi(S - I)$$

Integrate the transfer equation in momentum space

◆ radiation moment equations

$$\begin{aligned} \partial_t E_r + \partial_j F_r^j &= \rho \gamma \kappa (4\pi B - c E_r') - \rho \gamma (\kappa + \sigma) \frac{v_j \cdot F_r'^j}{c} \\ \frac{1}{c^2} \partial_t F_r^i + \partial_j P_r^{ij} &= \rho \gamma \kappa \frac{v^i}{c} \left(\frac{4\pi}{c} B - E_r' \right) \\ &\quad - \frac{\rho (\kappa + \sigma)}{\gamma + 1} \frac{u^i}{c} (u_j F_r'^j) - \frac{\rho (\kappa + \sigma)}{c} F_r^i \end{aligned}$$

κ : absorption coeff., σ_s : scattering coeff., B: Blackbody intensity
dash denotes the variables in the comoving frame.

$$\begin{aligned} E_r &= \frac{1}{c} \int d\nu d\Omega I && \text{radiation energy density} \\ F_r^i &= \int d\nu d\Omega I n^i && \text{radiation flux} \\ P_r^{ij} &= \frac{1}{c} \int d\nu d\Omega I n^i n^j && \text{radiation stress} \end{aligned}$$

(Takahashi+ '12)

How to close the equations?

HRT+ 12,
HRT & Ohsuga '12

assuming the equation of state of the radiation field:

$$P^{ij} = D^{ij}(E_r, \mathbf{F}_r) E_r$$

◆ Eddington app.

assuming the isotropic radiation pressure

$$P'^{ij} = \frac{\delta^{ij}}{3} E'_r$$

valid for optically thick medium.
radiation front propagates with velocity $c/\sqrt{3}$.

◆ M-1 closure

taking into account anisotropy of the radiation field

$$P^{ij} = \left[\frac{1 - \chi}{2} \delta^{ij} + \frac{3\chi - 1}{2} n^i n^j \right] E_r$$

(Levermore '84)

isotropic

anisotropic

$$\chi = \frac{3 + 4|\mathbf{f}|^2}{5 + 2\sqrt{4 - 3|\mathbf{f}|^2}}, \quad \mathbf{f} = \frac{\mathbf{F}_r}{cE_r}, \quad \mathbf{n} = \frac{\mathbf{F}_r}{|\mathbf{F}_r|}$$

shadow problem

irradiate an optically thick clump

the radiation field propagates in straight line with M-1 closure.

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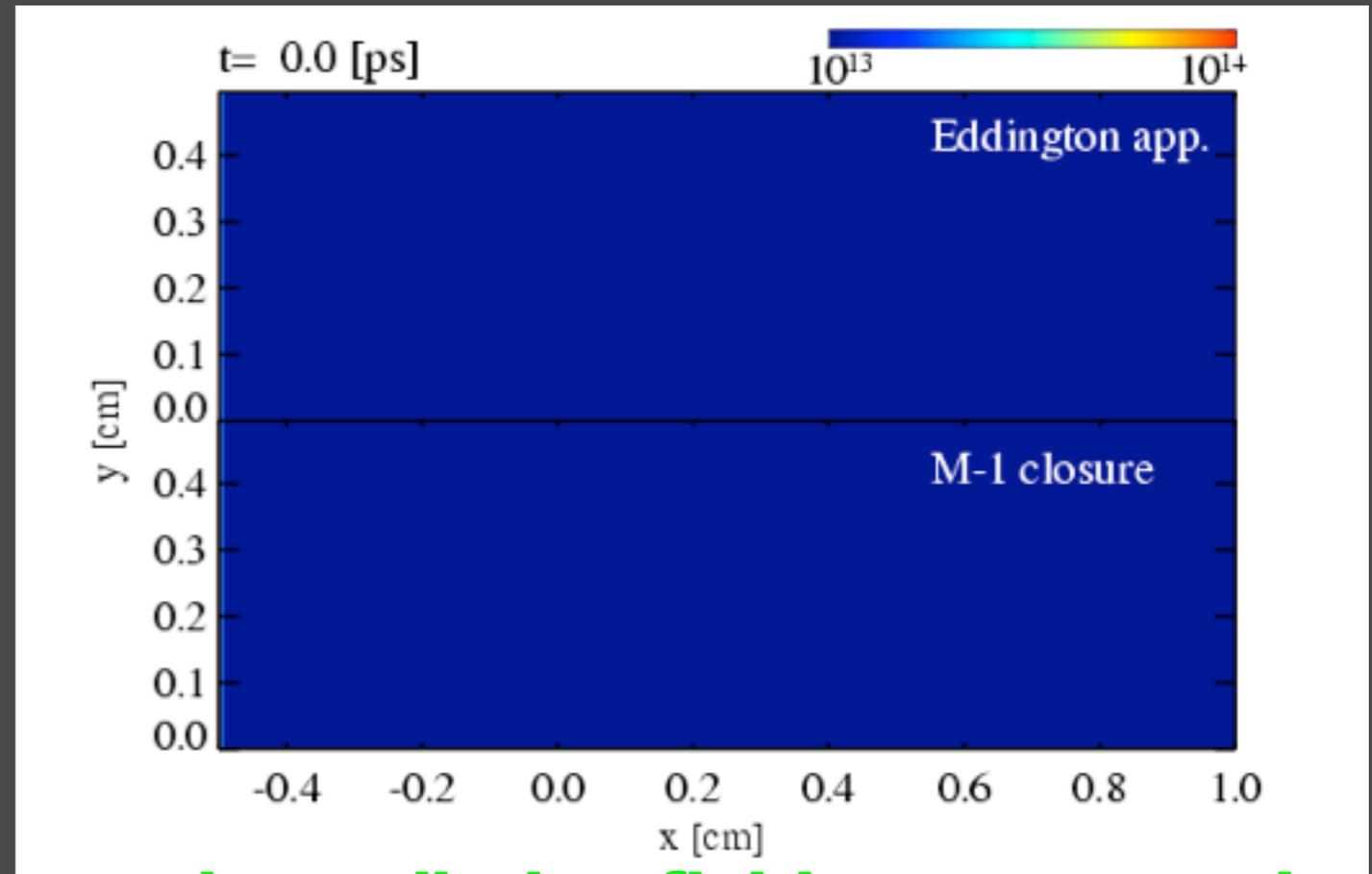
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Relativistic Resistive Radiation MHD(R3MHD)

mass conservation equation

$$\frac{\partial \rho \gamma}{\partial t} + \frac{\partial}{\partial x^\nu} (\rho \gamma v^\nu) = 0$$

gas energy conservation

$$\frac{\partial}{\partial t} [E_{\text{hydro}} + E_{\text{EM}}] + \nabla \cdot [m_{\text{hydro}} + m_{\text{MHD}}] = G^0$$

gas momentum equation

$$\frac{1}{c^2} \frac{\partial}{\partial t} [m_{\text{hydro}} + m_{\text{EM}}] + \nabla \cdot [P_{\text{hydro}} + P_{\text{MHD}}] = G$$

Maxwell equations

$$\frac{\partial \mathbf{B}}{\partial t} + c \nabla \times \mathbf{E} = 0 \quad \nabla \cdot \mathbf{E} = 4\pi q$$

$$\frac{\partial \mathbf{E}}{\partial t} - c \nabla \times \mathbf{B} = -4\pi \mathbf{j} \quad \nabla \cdot \mathbf{B} = 0$$

15 hyperbolic equation

Radiation moment equation

$$\frac{\partial E_r}{\partial t} + \nabla \cdot \mathbf{F}_r = -G^0$$

$$\frac{1}{c^2} \frac{\partial \mathbf{F}_r}{\partial t} + \nabla \cdot \mathbf{P}_r = -G$$

algebraic equations

gas : E.o.S.

rad. : M-1 closure

E.M. : Ohm's law

Petschek Type Reconnection in Uniformly Distributed Radiation Field

parameter:

density 1.0×10^{-2} g/cm³,

$T_{\text{gas}} 1 \times 10^8$ K,

$T_{\text{rad}} 1 \times 10^8$ K,

$B = 1 \times 10^{10}$ Gauss

$V_A = 0.69c$

$\beta = 4.1 \times 10^{-5}$

$\sigma = 0.89$

radiation process

abs.: free-free absorption (m.f.p. = 1.6×10^4 km)

scat.: electron scattering (m.f.p. = 2.5×10^{-3} km)

model

force-free Harris sheet

localized resistivity

radiation energy density

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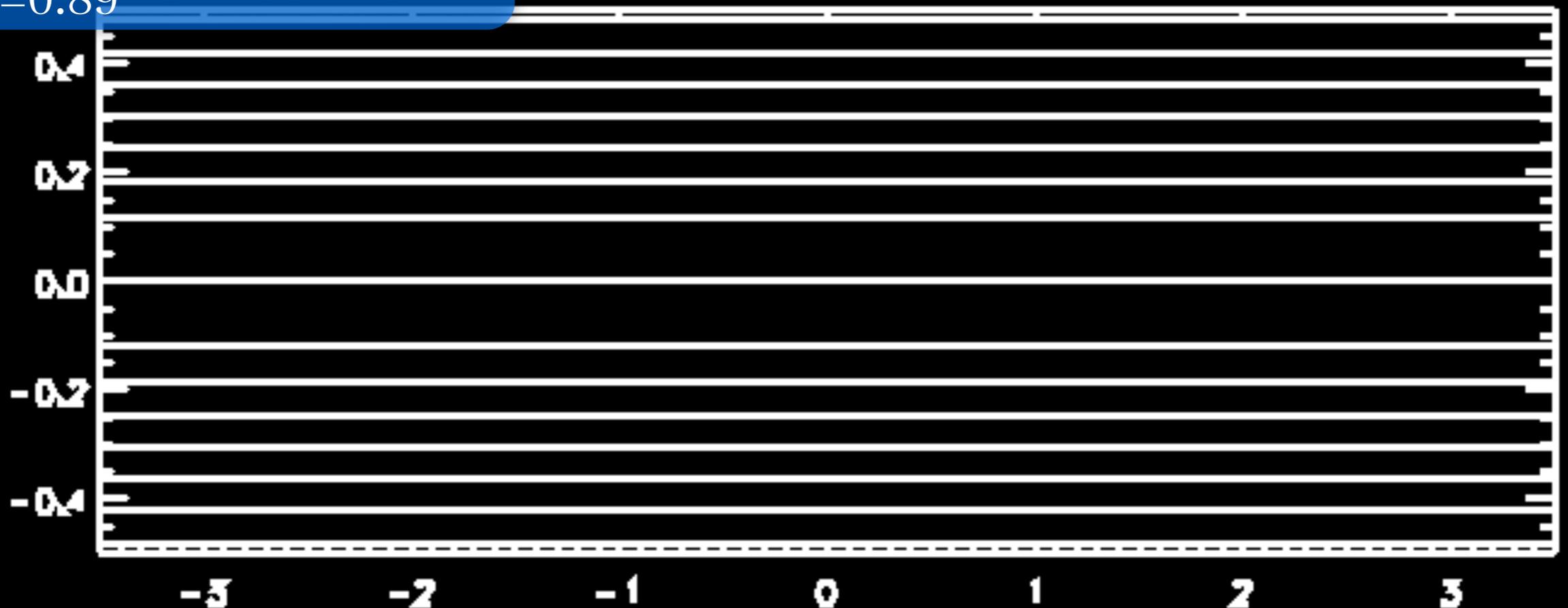
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model

force-free Harris sheet

localized resistivity

without radiation

with radiation

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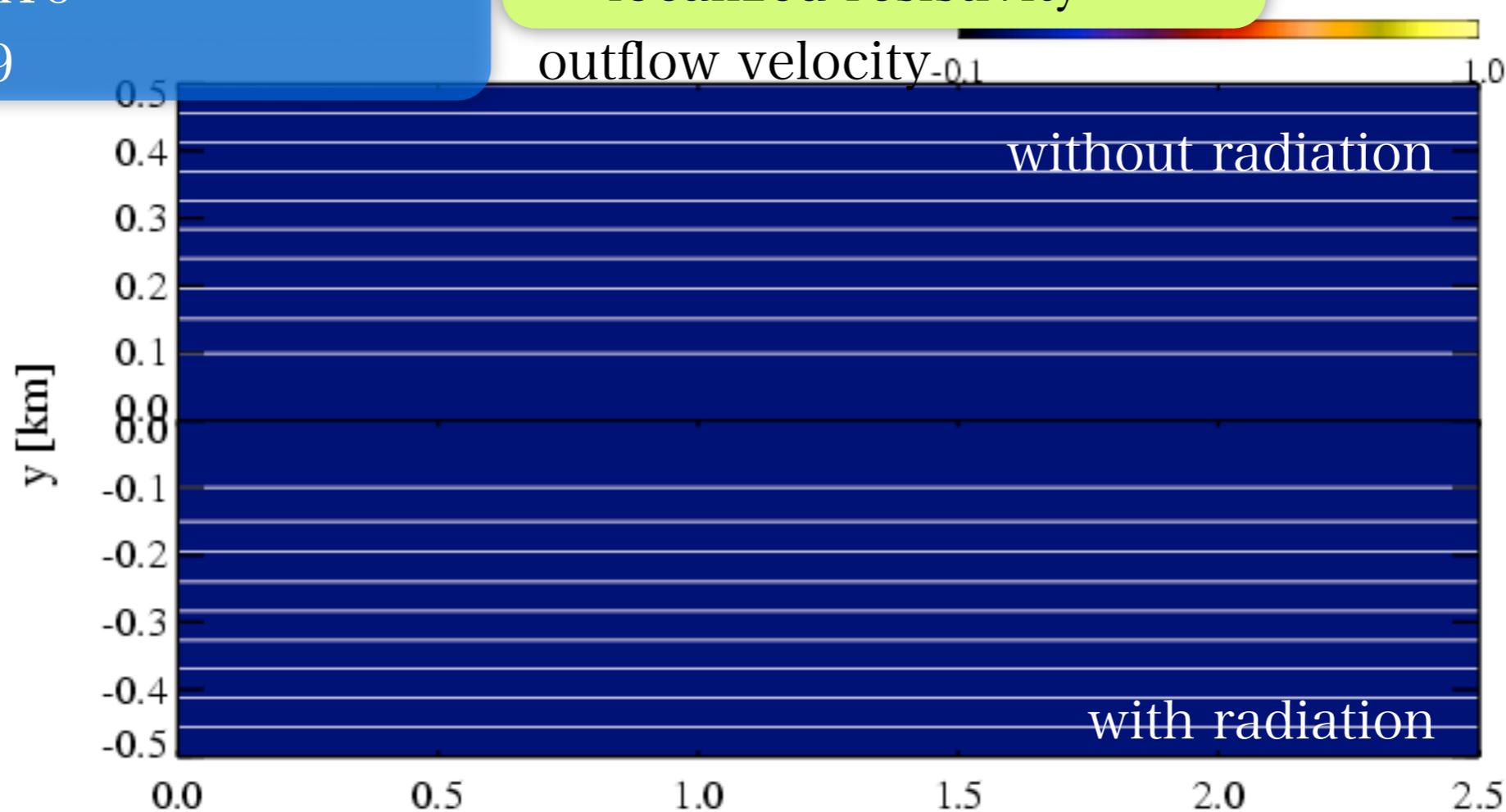
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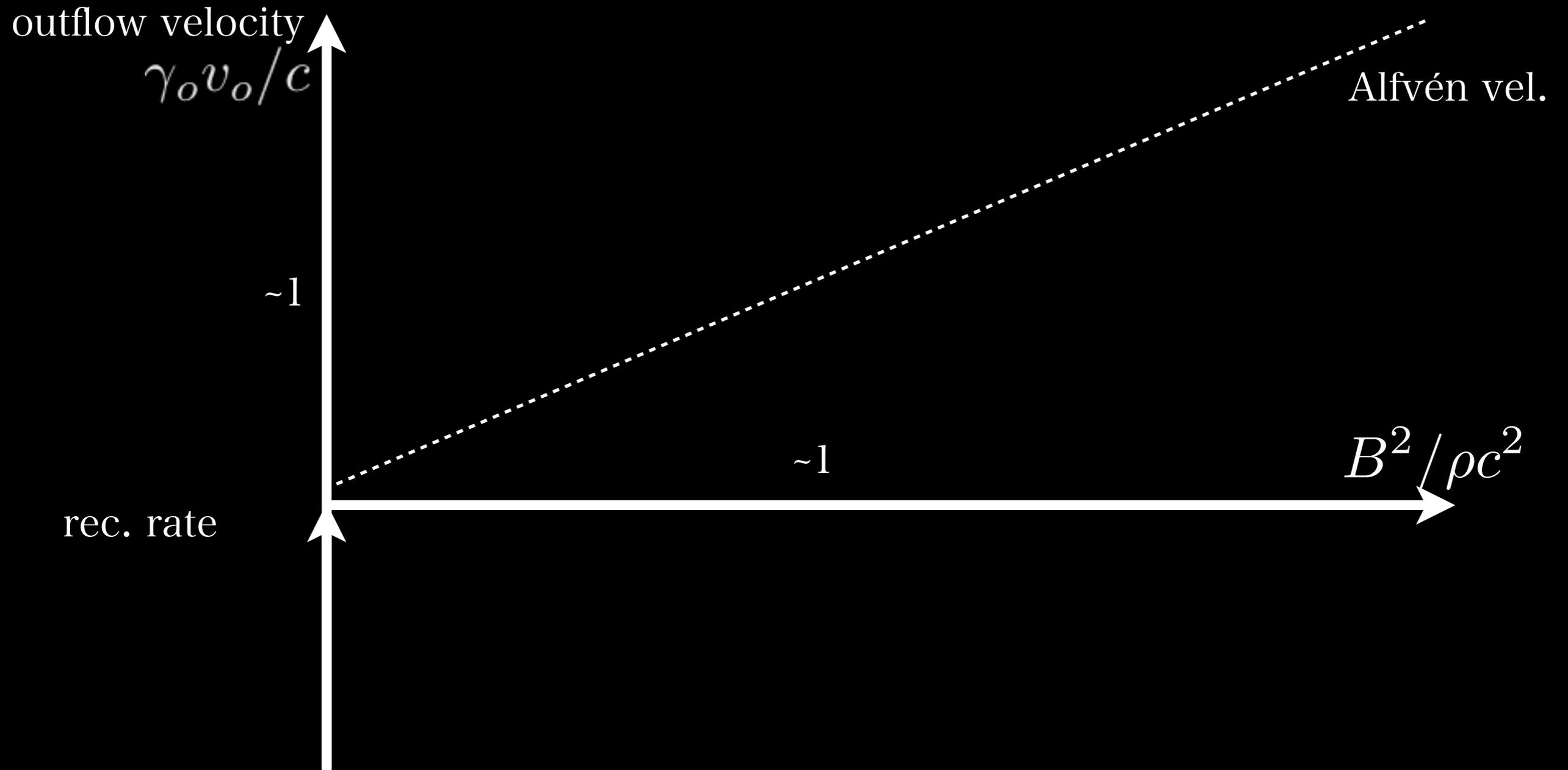
force-free Harris sheet

localized resistivity



Summary

We showed the first results of the MRX with R3MHD code

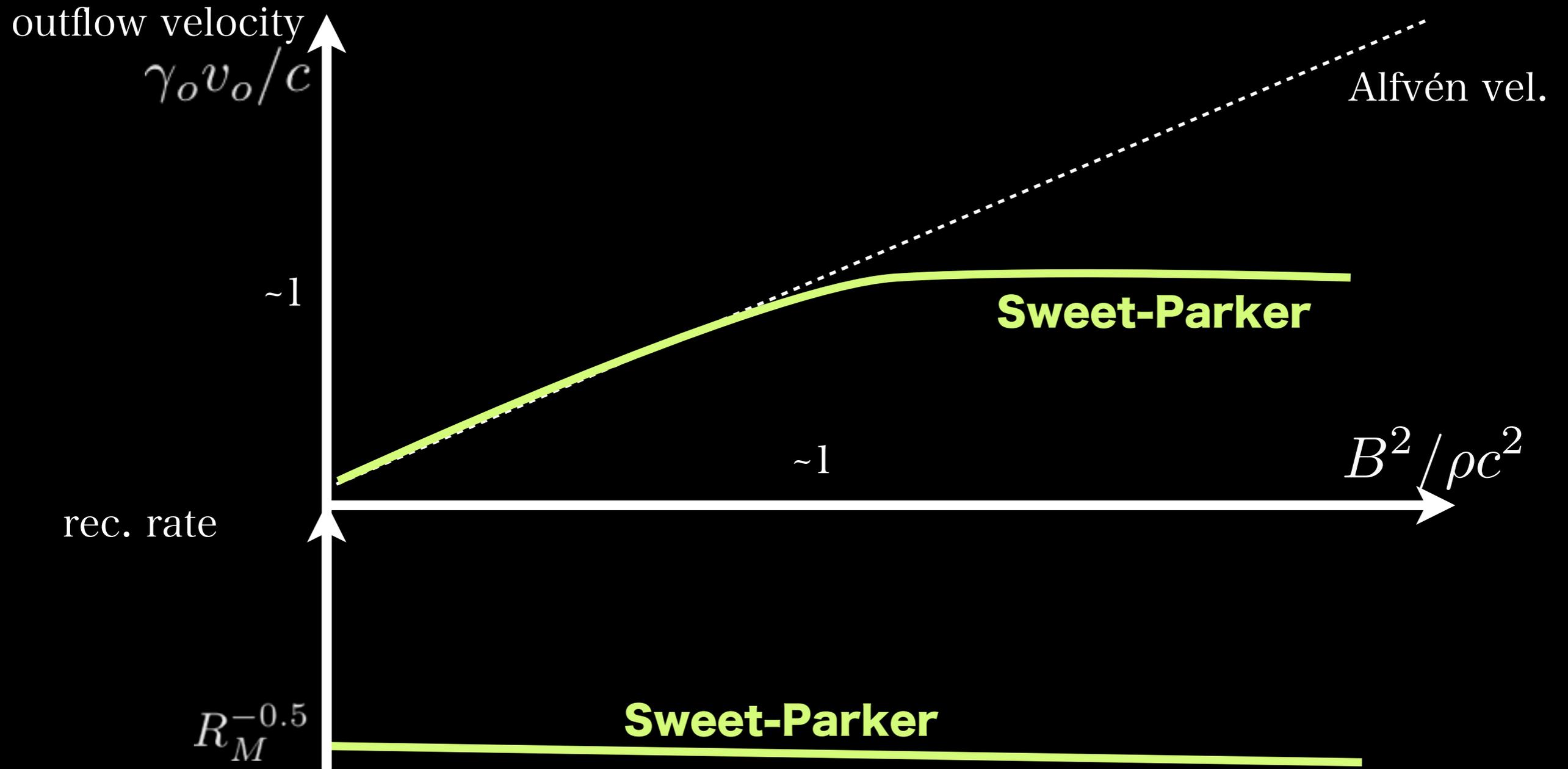


In the relativistic reconnection,

- outflow velocity decreases due to the radiative dragging force.
- reconnection rate decreases to balance the mass conservation.

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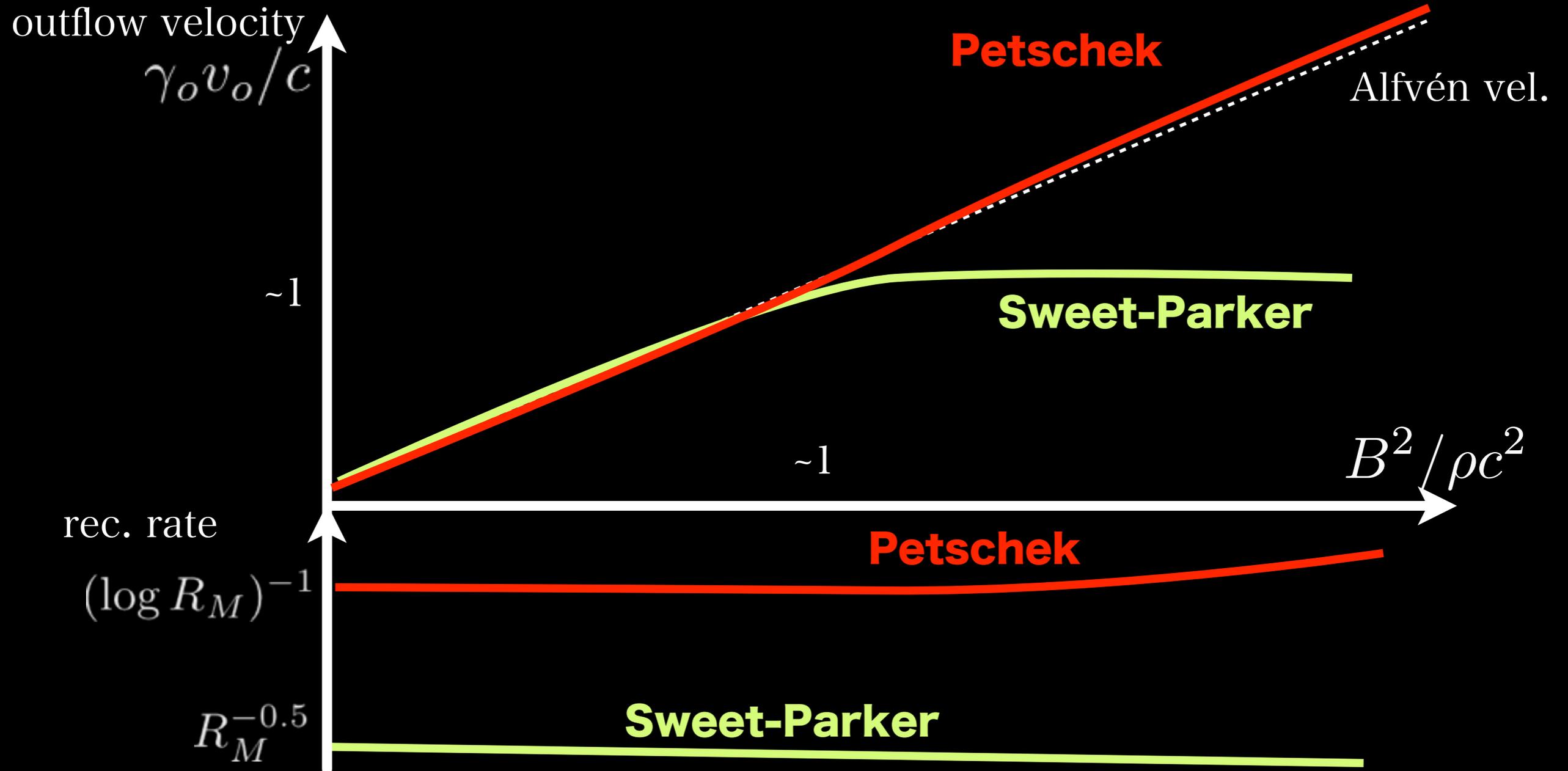


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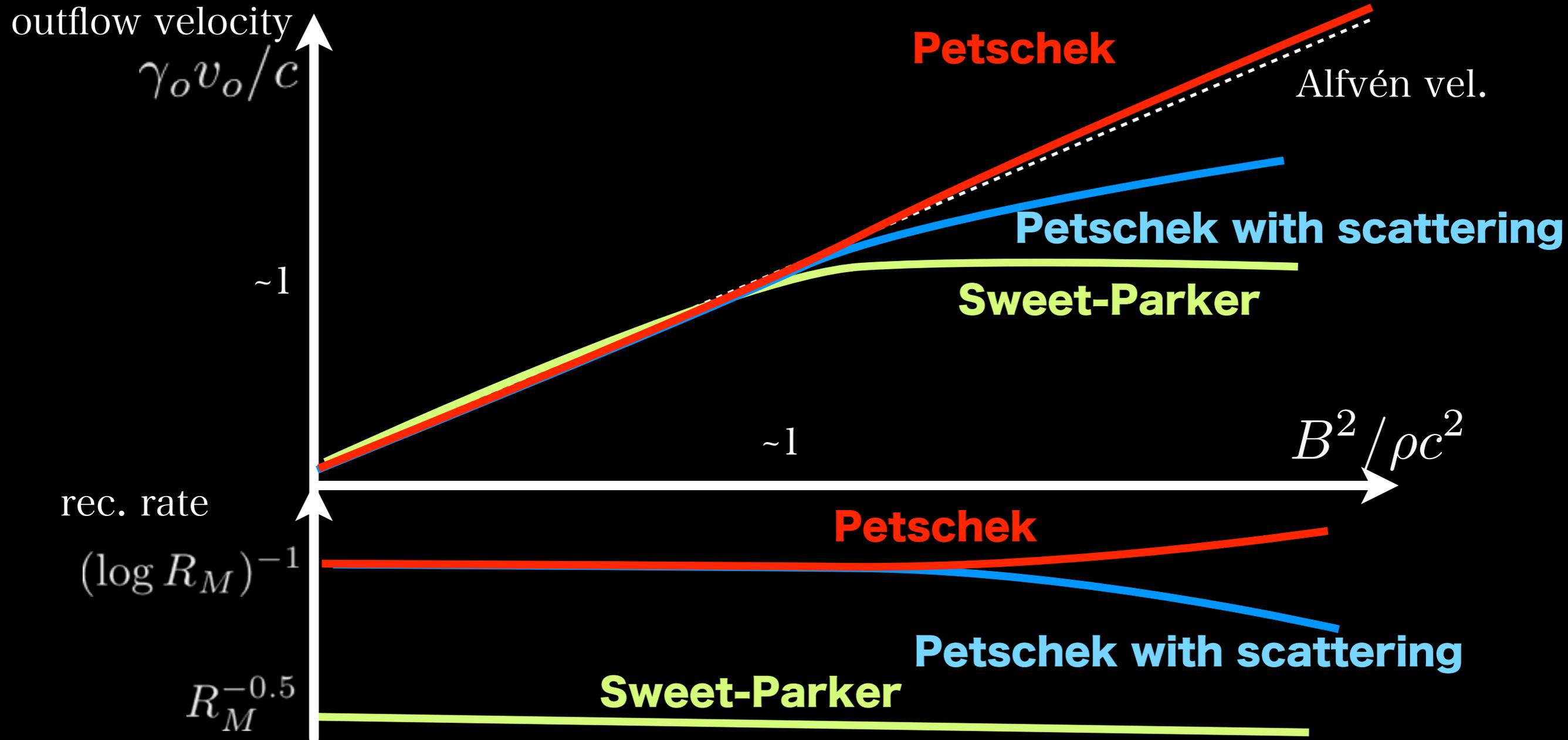


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Summary

We showed the first results of the MRX with R3MHD code.

Sweet-Parker type

- The outflow velocity is mildly relativistic ($\gamma \sim 1$).
- The reconnection rate is small: $\mathcal{R} = R_M^{-0.5}$

Petschek type

- The outflow velocity is relativistic ($\gamma = \sqrt{1 + \sigma}$).
- The reconnection rate is large $\mathcal{R} \simeq (\log R_M)^{-1}$

Petschek type with electron scattering

- The outflow velocity decreases by the radiation dragging force
- The reconnection rate also decreases.

We have to include the synchrotron cooling effects, which is the dominant source for the opacity in the relativistic plasma (Uzdensky & McKinney '11).

We should perform the R3MHD simulation including the synchrotron radiation to construct a more realistic model.